deep image retrieval and manifold learning

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Inria Rennes-Bretagne Atlantique

Paris, March 2018

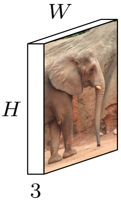
outline

feature pooling
manifold learning
fine-tuning
ranking on manifolds
fast spectral ranking
mining on manifolds

feature pooling

image ranking by CNN features

[Krizhevsky et al. 2012]



- 3-channel RGB input, 224×224
- AlexNet pre-trained on ImageNet for classification
- last fully connected layer (fc_6): global descriptor of dimension $k = 4096$

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image ranking by CNN features

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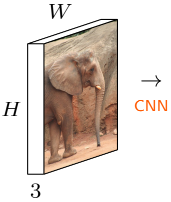

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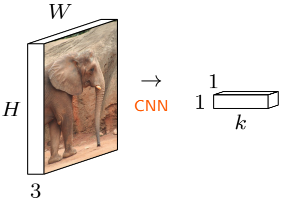

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- query images
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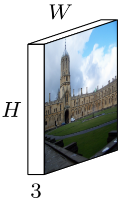
neural codes for image retrieval

[Babenko et al. 2014]



- 3-channel RGB input, 224×224
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- alternatively: fully connected layers $\mathrm{fc}_6, \mathrm{fc}_7$, global descriptors of dimension $k' = 4096$ (best is fc_6)
- in each case: PCA-whitening, ℓ_2 normalization

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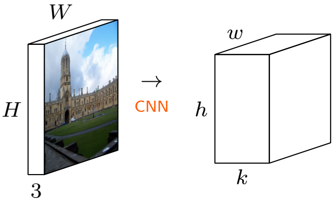

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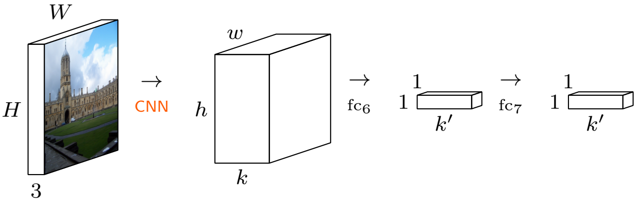

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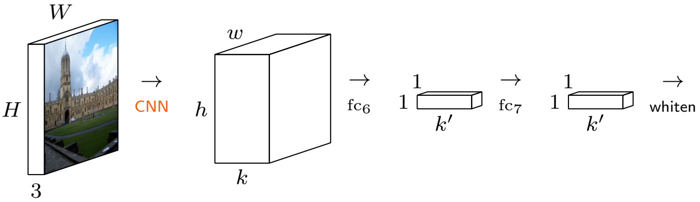

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neural codes for image retrieval

- fine-tuning by softmax on 672 classes of 200k landmark photos
- outperforms VLAD and Fisher vectors on standard retrieval benchmarks, but still inferior to SIFT local descriptors

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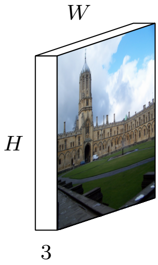
regional CNN features

[Razavian et al. 2015]



- 3-channel RGB input, largest square region extracted
- fixed multiscale overlapping regions, warped into $w \times h = 227 \times 227$
- each region yields a $w' \times h' \times k = 36 \times 36 \times 256$ dimensional feature at the last convolutional layer of AlexNet
- global spatial max-pooling
- ℓ_2-normalization, PCA-whitening of each descriptor

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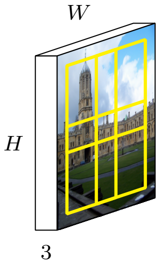

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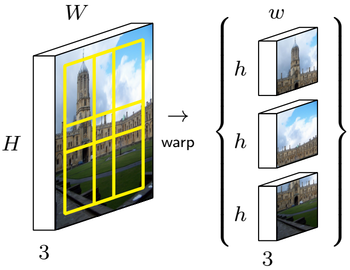

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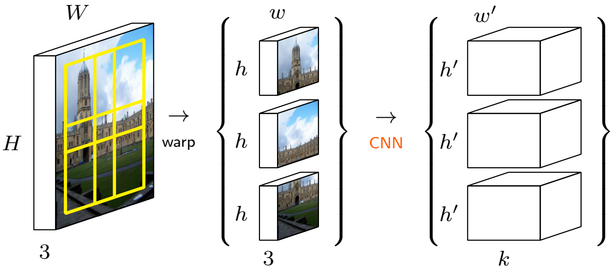

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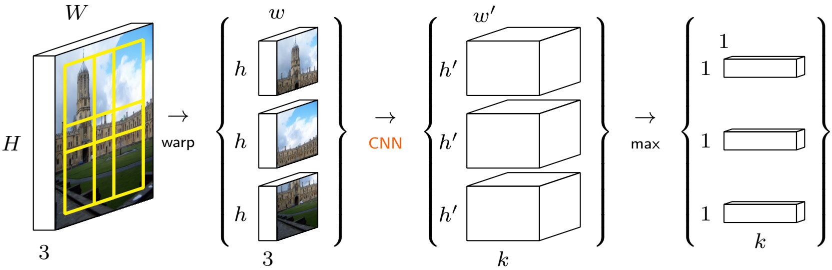

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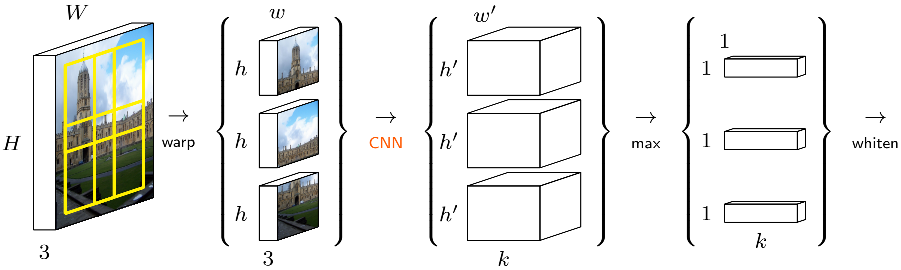

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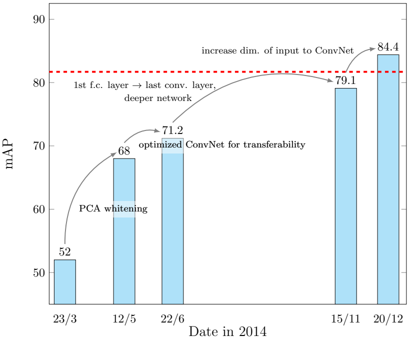

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- however, this is based on multiple regional descriptors per image and exhaustive pairwise matching of all descriptors of query and all dataset images, which is not practical

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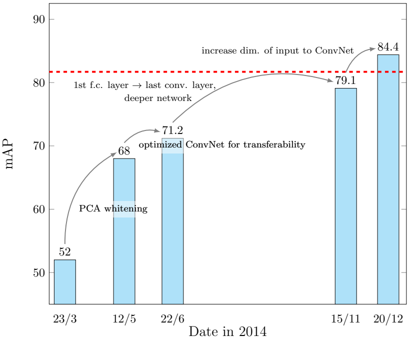

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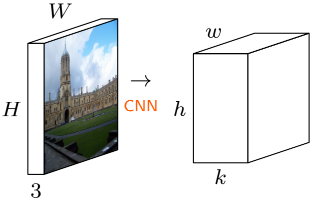
regional max-pooling (R-MAC)

[Tolias et al. 2016]



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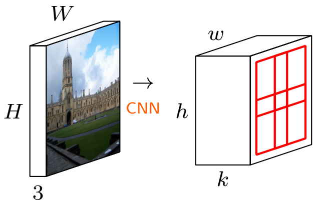

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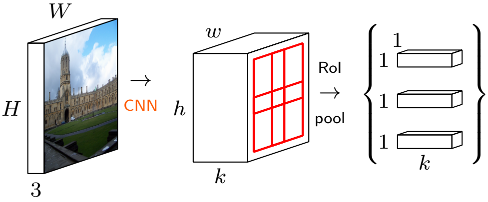

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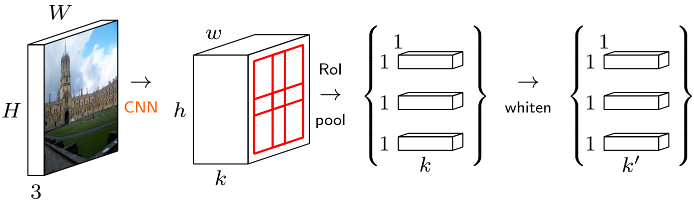

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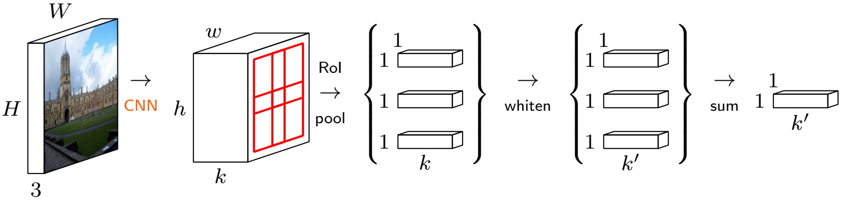

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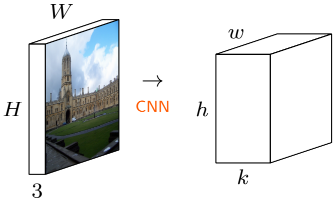
global max-pooling (MAC)



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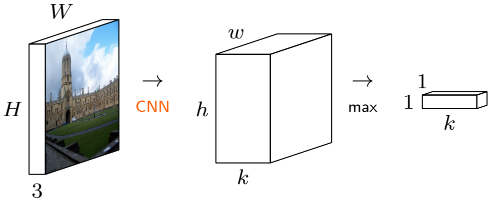

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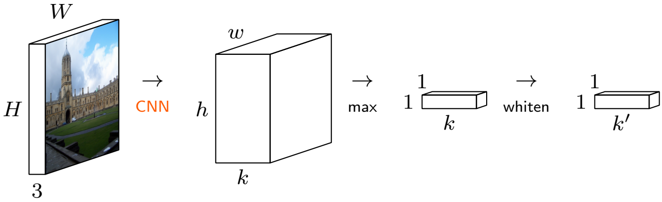

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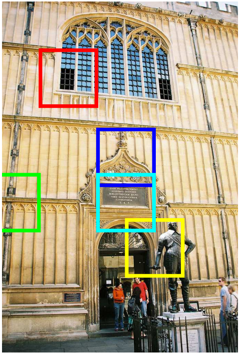
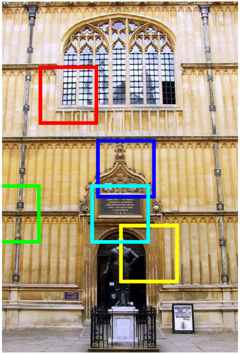
global max-pooling: matching





- receptive fields of 5 components of MAC vectors that contribute most to image similarity

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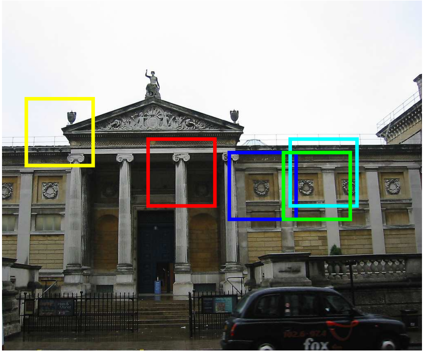

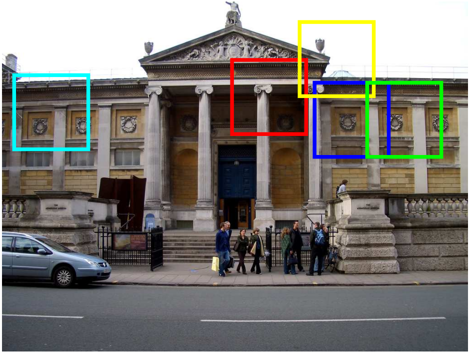

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manifold learning

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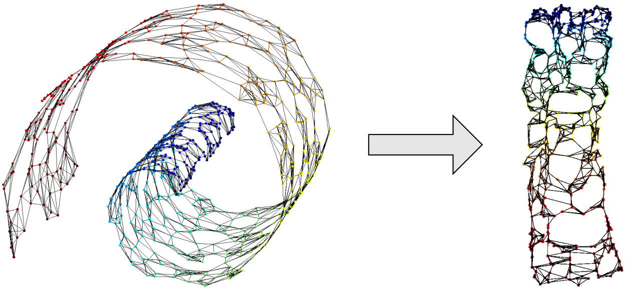

- *e.g.* Isomap: apply PCA to the geodesic (graph) distance matrix
- *e.g.* kernel PCA: apply PCA to the Gram matrix of a nonlinear kernel
- other topology-preserving methods are only focusing on distances to nearest neighbors
- many classic methods use eigenvalue decomposition and most do not learn and explicit mapping from the input to the embedding space

siamese architecture

[Chopra et al. 2005]

$\mathbf{x}_i$ $\mathbf{x}_j$

- an input sample is a pair $(\mathbf{x}_i, \mathbf{x}_j)$
- both $\mathbf{x}_i, \mathbf{x}_j$ go through the same function f with shared parameters θ
- loss ℓ_{ij} is measured on output pair $(\mathbf{y}_i, \mathbf{y}_j)$ and target t_{ij}

Chopra, Hadsell, Lecun, CVPR 2005. Learning a Similarity Metric Discriminatively, with Application to Face Verification.

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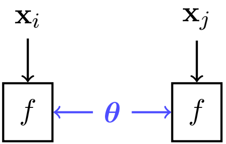

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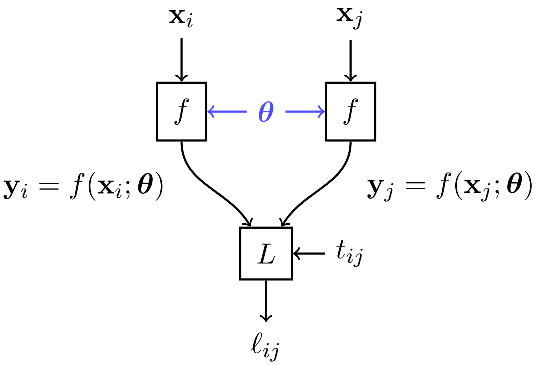

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contrastive loss

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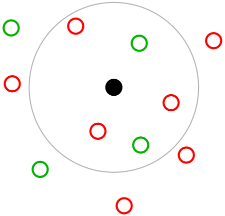

- input samples $\mathbf{x}_i$, output vectors $\mathbf{y}_i = f(\mathbf{x}_i; \boldsymbol{\theta})$
- target variables $t_{ij} = \mathbb{1}[\mathrm{sim}(\mathbf{x}_i, \mathbf{x}_j)]$
- contrastive loss is a function of distance $\|\mathbf{y}_i - \mathbf{y}_j\|$ only

$$\ell_{ij} = L((\mathbf{y}_i, \mathbf{y}_j), t_{ij}) = \ell(\|\mathbf{y}_i - \mathbf{y}_j\|, t_{ij})$$

- similar samples are attracted

$$\ell(x, t) = t\ell^+(x) + (1-t)\ell^-(x) = tx^2 + (1-t)[m-x]_+^2$$

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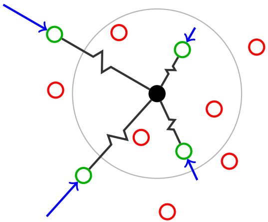

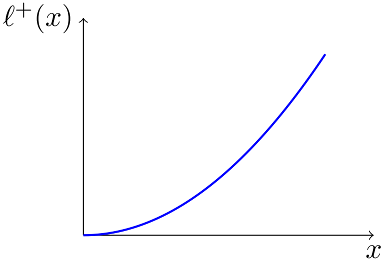

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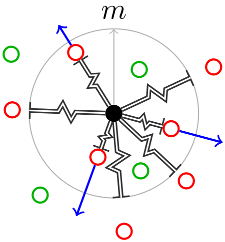

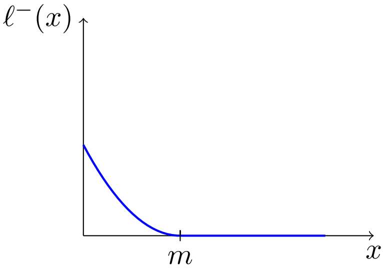

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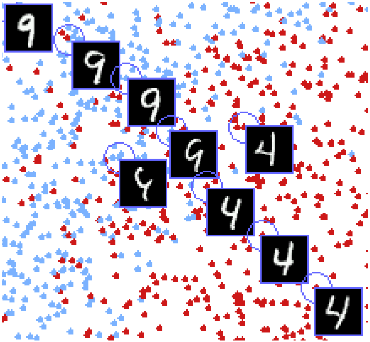
- dissimilar samples are repelled if closer than margin m

$$\ell(x,t) = t\ell^+(x) + (1-t)\boxed{\ell^-(x)} = tx^2 + (1-t)\boxed{[m-x]_+^2}$$

Hadsell, Chopra, Lecun. CVPR 2006. Dimensionality Reduction By Learning an Invariant Mapping.

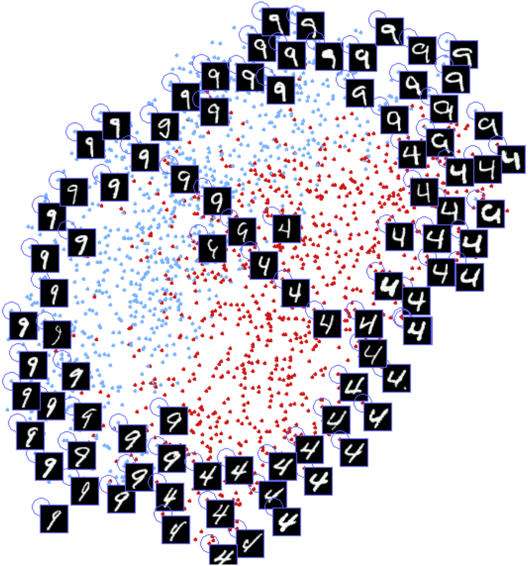
manifold learning: MNIST



- 3k samples of each of digits $4, 9$
- each sample similar to its 5 Euclidean nearest neighbors, and dissimilar to all other points
- 30k similar pairs, 18M dissimilar pairs

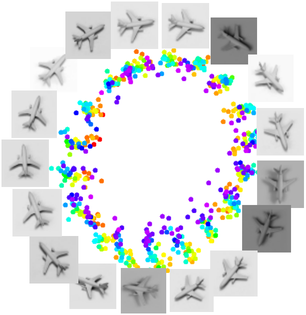
Hadsell, Chopra, Lecun. CVPR 2006. Dimensionality Reduction By Learning an Invariant Mapping.

manifold learning: MNIST



Hadsell, Chopra, Lecun. CVPR 2006. Dimensionality Reduction By Learning an Invariant Mapping.

manifold learning: NORB



- 972 images of airplane class: 18 azimuths (every 20°), 9 elevations (in $[30^\circ, 70^\circ]$, every 5°), 6 lighting conditions
- samples similar if taken from contiguous azimuth or elevation, regardless of lighting
- 11k similar pairs, 206M dissimilar pairs
- cylindrer in 3d: azimuth on circumference, elevation on height

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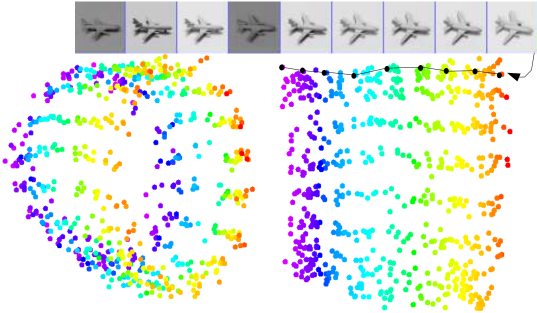

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triplet architecture

[Wang et al. 2014]

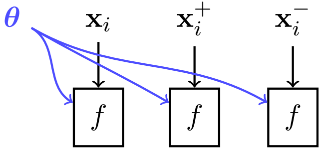
$\mathbf{x}_i \qquad \mathbf{x}_i^+ \qquad \mathbf{x}_i^-$

- an input sample is a triplet $(\mathbf{x}_i, \mathbf{x}_i^+, \mathbf{x}_i^-)$
- $\mathbf{x}_i, \mathbf{x}_i^+, \mathbf{x}_i^-$ go through the same function f with shared parameters $\boldsymbol{\theta}$
- loss ℓ_i measured on output triplet $(\mathbf{y}_i, \mathbf{y}_i^+, \mathbf{y}_i^-)$

Wang, Song, Leung, Rosenberg, Wang, Philbin, Chen, Wu. CVPR 2014. Learning Fine-Grained Image Similarity with Deep Ranking.

triplet architecture

[Wang et al. 2014]



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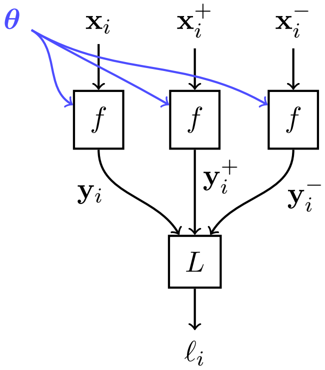

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Wang, Song, Leung, Rosenberg, Wang, Philbin, Chen, Wu. CVPR 2014. Learning Fine-Grained Image Similarity with Deep Ranking.

triplet loss

- input "anchor" $\mathbf{x}_i$, output vector $\mathbf{y}_i = f(\mathbf{x}_i; \boldsymbol{\theta})$
- positive $\mathbf{y}_i^+ = f(\mathbf{x}_i^+; \boldsymbol{\theta})$, negative $\mathbf{y}_i^- = f(\mathbf{x}_i^-; \boldsymbol{\theta})$
- triplet loss is a function of distances $\|\mathbf{y}_i - \mathbf{y}_i^+\|, \|\mathbf{y}_i - \mathbf{y}_i^-\|$ only

$$\ell_i = L(\mathbf{y}_i, \mathbf{y}_i^+, \mathbf{y}_i^-) = \ell(\|\mathbf{y}_i - \mathbf{y}_i^+\|, \|\mathbf{y}_i - \mathbf{y}_i^-\|)$$

$$\ell(x^+, x^-) = \left[m + (x^+)^2 - (x^-)^2\right]_+$$

so distance $\|\mathbf{y}_i - \mathbf{y}_i^+\|$ should be less than $\|\mathbf{y}_i - \mathbf{y}_i^-\|$ by margin m

- by taking two pairs $(\mathbf{x}_i, \mathbf{x}_i^+)$ and $(\mathbf{x}_i, \mathbf{x}_i^-)$ at a time with targets $1, 0$ respectively, the contrastive loss can be written similarly

$$\ell(x^+, x^-) = (x^+)^2 + \left[m - x^-\right]_+^2$$

so distance $\|\mathbf{y}_i - \mathbf{y}_i^+\|$ should small and $\|\mathbf{y}_i - \mathbf{y}_i^-\|$ larger than m

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unsupervised learning by context prediction

[Doersch et al. 2015]

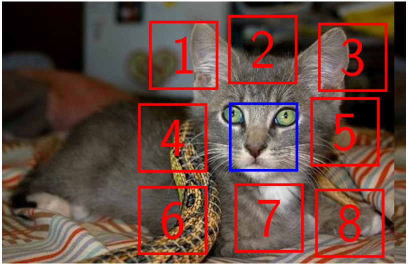
- sample random pairs of patches in one of eight spatial configurations
- patches are randomly jittered and do not overlap
- like solving a puzzle, learn to predict the relative position

$$f\left(\quad , \quad \right) = 3$$

Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

unsupervised learning by context prediction

[Doersch et al. 2015]



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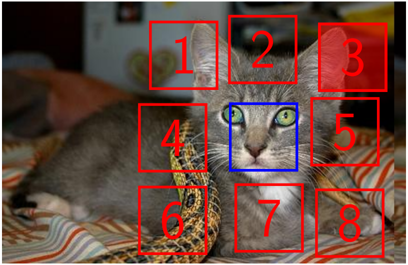

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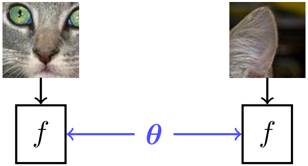
Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

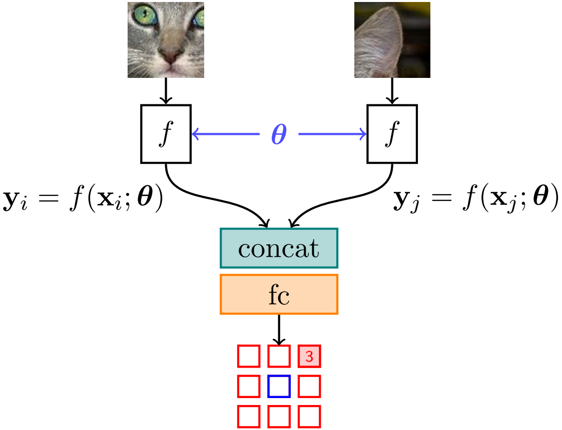
context prediction: architecture



- network f learned by siamese architecture
- representations are concatenated and followed by softmax classifier, where each spatial configuration is a class

Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

context prediction: architecture



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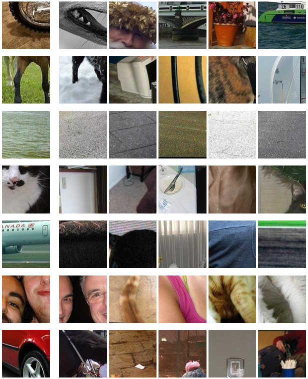
Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

context prediction: examples

- input image
- nearest neighbors with randomly initialized network
- trained by supervised classification on ImageNet
- unsupervised training from scratch on the context prediction task

Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

context prediction: examples



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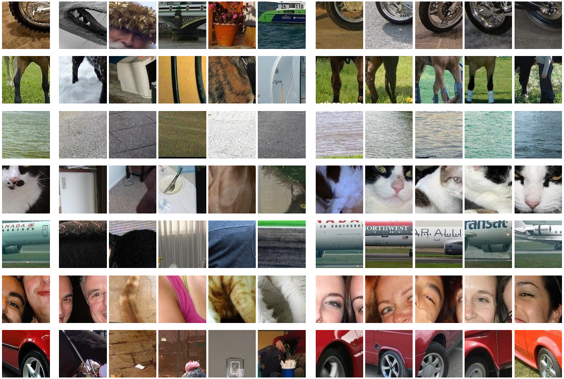

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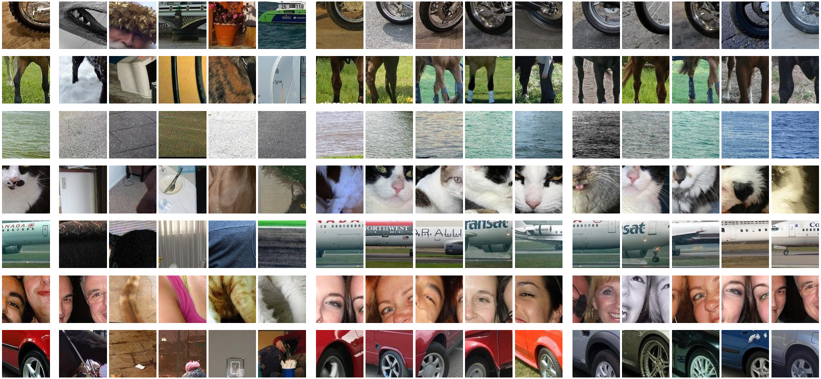

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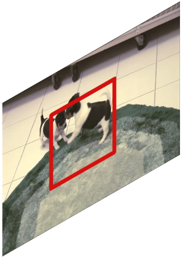
Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

unsupervised learning on video: tracking

[Wang et al. 2015]

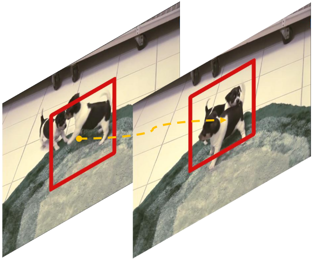


- estimate motion and find the region that contains most motion
- track this region for a number of frames
- generate a pair of matching patches on the first and last frames

Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

unsupervised learning on video: tracking

[Wang et al. 2015]



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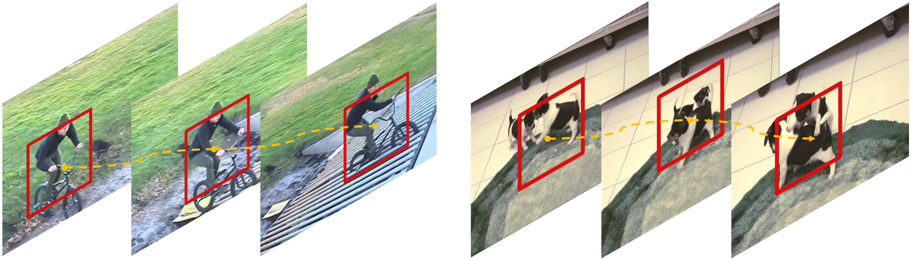

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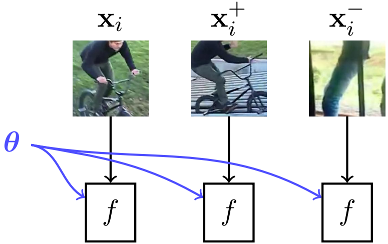
Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

unsupervised learning on video: architecture

- input query $\mathbf{x}_i$ (first frame), tracked $\mathbf{x}_i^+$ (last frame), random $\mathbf{x}_i^-$
- $\mathbf{x}_i, \mathbf{x}_i^+, \mathbf{x}_i^-$ go through the same function f with shared parameters $\boldsymbol{\theta}$
- triplet loss ℓ_i measured on output triplet $(\mathbf{y}_i, \mathbf{y}_i^+, \mathbf{y}_i^-)$

Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

unsupervised learning on video: architecture



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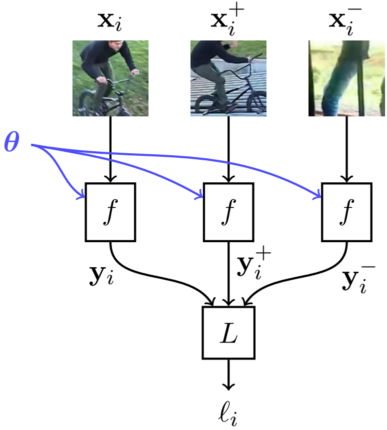

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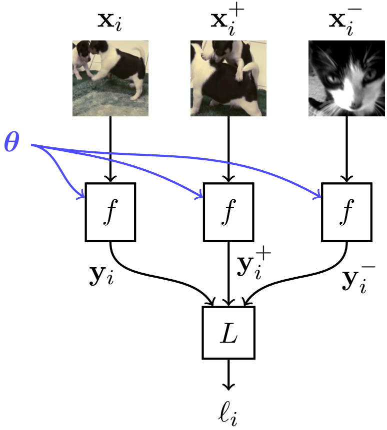

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Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

unsupervised learning on video: objective

$\Big\| f\Big($  $\Big) - f\Big($  $\Big)\Big\|^2 < \Big\| f\Big($  $\Big) - f\Big($  $\Big)\Big\|^2 - m$

$\Big\| f\Big($  $\Big) - f\Big($  $\Big)\Big\|^2 < \Big\| f\Big($  $\Big) - f\Big($  $\Big)\Big\|^2 - m$

- so, the objective is that squared distance $\|\mathbf{y}_i - \mathbf{y}_i^+\|^2$ is less than $\|\mathbf{y}_i - \mathbf{y}_i^-\|^2$ by margin m

Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

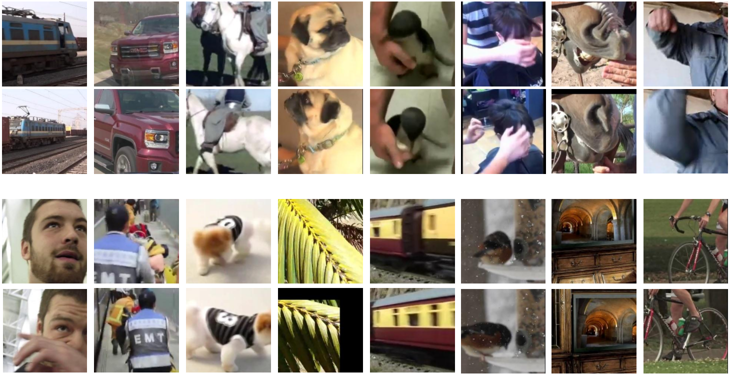
unsupervised learning on video: more examples

$\mathbf{x}_i$

$\mathbf{x}_i^+$

$\mathbf{x}_i$

$\mathbf{x}_i^+$

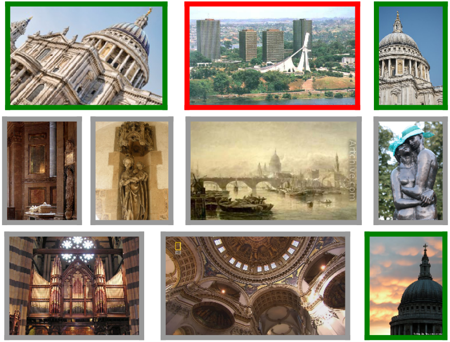


- input query $\mathbf{x}_i$ (first frame), tracked $\mathbf{x}_i^+$ (last frame)

fine-tuning

deep image retrieval: dataset cleaning

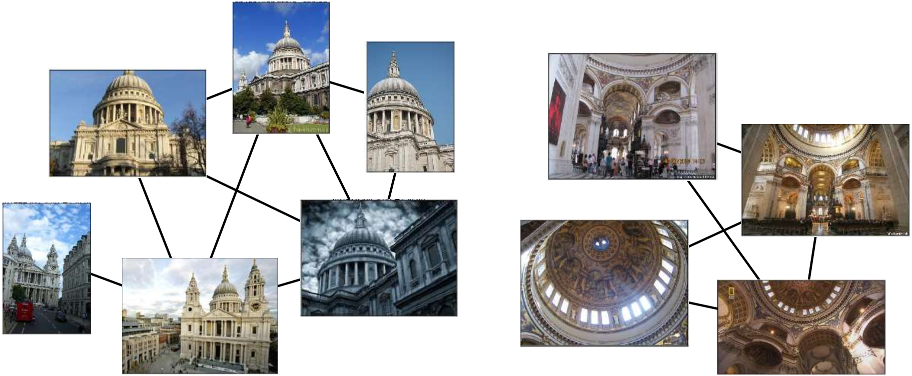
[Gordo et al. 2016]



- start from landmark dataset (192k images) and clean it (49k images)
- use it to fine-tune a network pre-trained on ImageNet for classification
- prototypical, non-prototypical and incorrect images per class
- only prototypical are kept to reduce intra-class variability

Gordo, Almazan, Revaud, Larlus. ECCV 2016. Deep Image Retrieval: Learning Global Representations for Image Search.

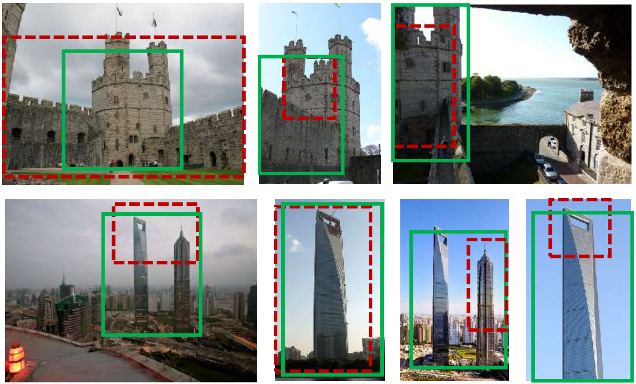
deep image retrieval: prototypical views



- pairwise match images per class by SIFT descriptors and fast spatial matching
- connect images into a graph and compute the connected components
- keep only the largest component

Gordo, Almazan, Revaud, Larlus. ECCV 2016. Deep Image Retrieval: Learning Global Representations for Image Search.

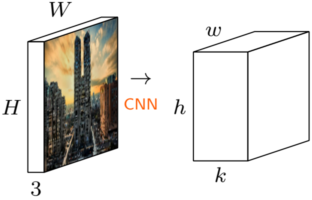
deep image retrieval: bounding boxes



- automatically find object bounding boxes
  - initialize with inlier features per image
  - update such that boxes are consistent over all matching pairs
- use bounding boxes to train a region proposal network

Gordo, Almazan, Revaud, Larlus. ECCV 2016. Deep Image Retrieval: Learning Global Representations for Image Search.

deep image retrieval: network, regions, pooling



- VGG-16 or ResNet-101 feature maps
- proposals detected on feature maps by RPN and max-pooled
- ℓ_2-normalization, PCA-whitening (FC layer), ℓ_2-normalization
- sum-pooling, ℓ_2-normalization (as in R-MAC)

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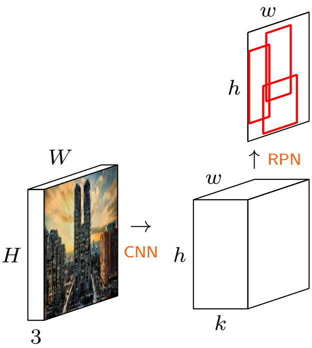
deep image retrieval: network, regions, pooling



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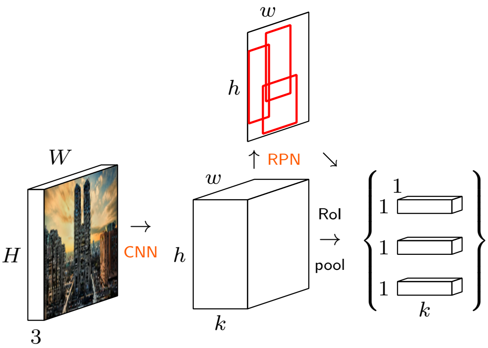

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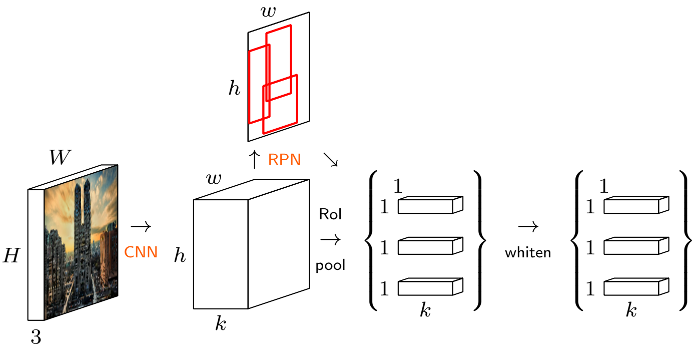

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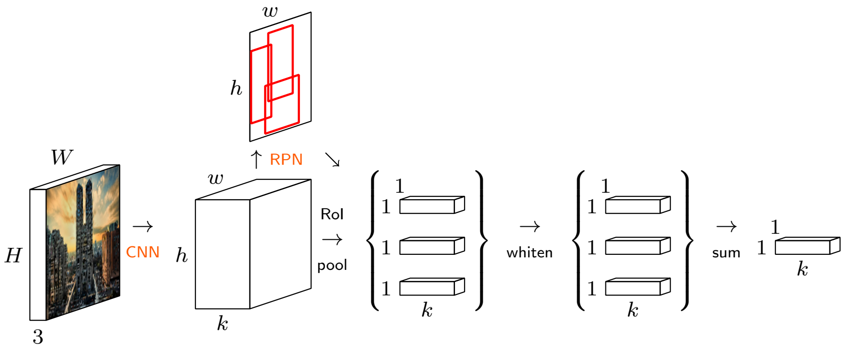

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Gordo, Almazan, Revaud, Larlus. ECCV 2016. Deep Image Retrieval: Learning Global Representations for Image Search.

deep image retrieval: architecture

$\mathbf{x}_i$

$\mathbf{x}_i^+$

$\mathbf{x}_i^-$

- query $\mathbf{x}_i$, relevant $\mathbf{x}_i^+$ (same building), irrelevant $\mathbf{x}_i^-$ (other building)
- $\mathbf{x}_i, \mathbf{x}_i^+, \mathbf{x}_i^-$ go through function f including features, RPN, pooling
- triplet loss ℓ_i measured on output $(\mathbf{y}_i, \mathbf{y}_i^+, \mathbf{y}_i^-)$

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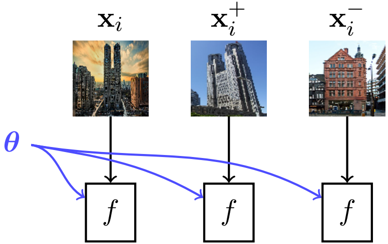
deep image retrieval: architecture



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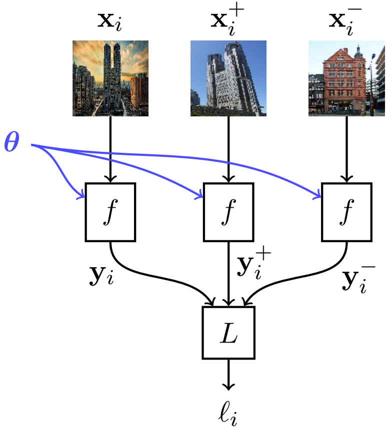
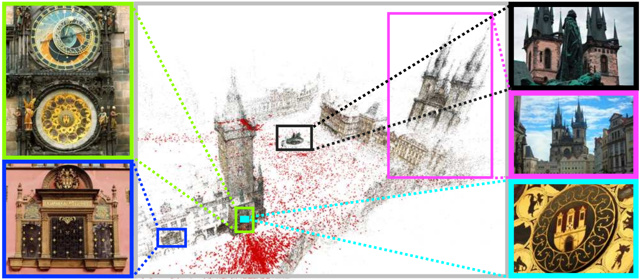


- query $\mathbf{x}_i$, relevant $\mathbf{x}_i^+$ (same building), irrelevant $\mathbf{x}_i^-$ (other building)
- $\mathbf{x}_i, \mathbf{x}_i^+, \mathbf{x}_i^-$ go through function f including features, RPN, pooling
- triplet loss ℓ_i measured on output $(\mathbf{y}_i, \mathbf{y}_i^+, \mathbf{y}_i^-)$

Gordo, Almazan, Revaud, Larlus. ECCV 2016. Deep Image Retrieval: Learning Global Representations for Image Search.

learning from bag-of-words: 3d reconstruction

[Radenovic et al. 2016]



- start from an independent dataset of 7.4M images, no class labels
- clustering, pairwise matching and reconstruction of 713 3d models containing 165k unique images
- 3d models playing the same role as classes in deep image retrieval
- again, fine-tune a network pre-trained on ImageNet for classification

Radenovic, Tolias, Chum. ECCV 2016. CNN Image Retrieval Learns From BoW: Unsupervised Fine-Tuning with Hard Examples.
Schönberger, Radenovic, Chum and Frahm. CVPR 2015. From Single Image Query to Detailed 3D Reconstruction.

learning from bag-of-words: positive pairs








- input query
- positive images found in same model by minimum MAC distance, maximum inliers, or drawn at random from images having at least a given number of inliers (more challenging)

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Radenovic, Tolias, Chum. ECCV 2016. CNN Image Retrieval Learns From BoW: Unsupervised Fine-Tuning with Hard Examples.

learning from bag-of-words: negative pairs

- input query
- negative images found in different models
- hard negatives are most similar to query, *i.e.* with minimum MAC distance
- hardest negative, nearest neighbors from all other models, or nearest neighbors, one per model (higher variability)

learning from bag-of-words: negative pairs

- input query
- negative images found in different models
- hard negatives are most similar to query, *i.e.* with minimum MAC distance
- hardest negative, nearest neighbors from all other models, or nearest neighbors, one per model (higher variability)

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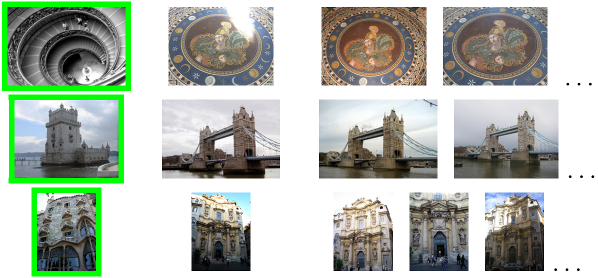
learning from bag-of-words: negative pairs



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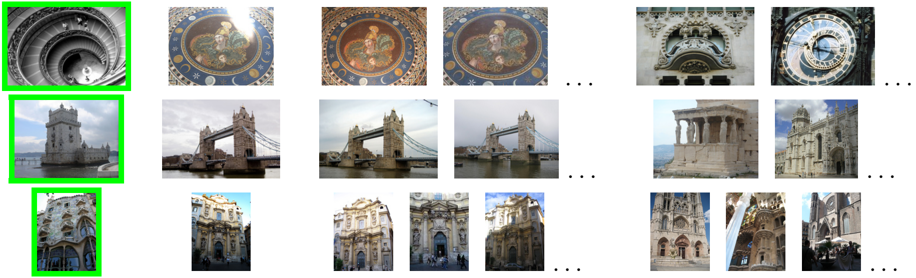

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Radenovic, Tolias, Chum. ECCV 2016. CNN Image Retrieval Learns From BoW: Unsupervised Fine-Tuning with Hard Examples.

learning from bag-of-words: architecture

$\mathbf{x}_i$ $\mathbf{x}_j$

- input $(\mathbf{x}_i, \mathbf{x}_j)$ of relevant or irrelevant images
- both $\mathbf{x}_i, \mathbf{x}_j$ go through function f including features and MAC pooling
- contrastive loss ℓ_{ij} measured on output $(\mathbf{y}_i, \mathbf{y}_j)$ and target t_{ij}

Radenovic, Tolias, Chum. ECCV 2016. CNN Image Retrieval Learns From BoW: Unsupervised Fine-Tuning with Hard Examples.

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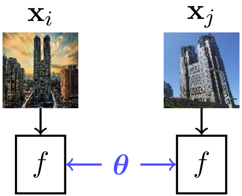

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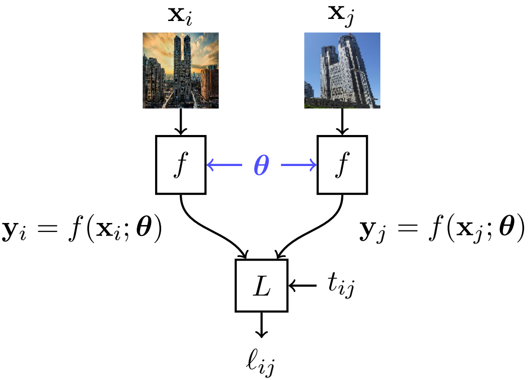

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- both $\mathbf{x}_i, \mathbf{x}_j$ go through function f including features and MAC pooling
- contrastive loss ℓ_{ij} measured on output $(\mathbf{y}_i, \mathbf{y}_j)$ and target t_{ij}

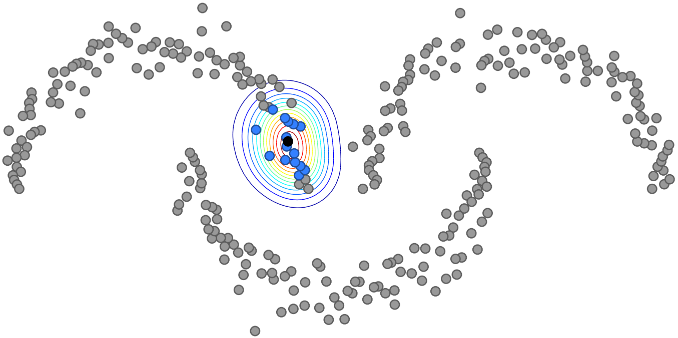
ranking on manifolds

ranking on manifolds: single query

- data points (•), query point (•), nearest neighbors (•)
- iteration × 30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

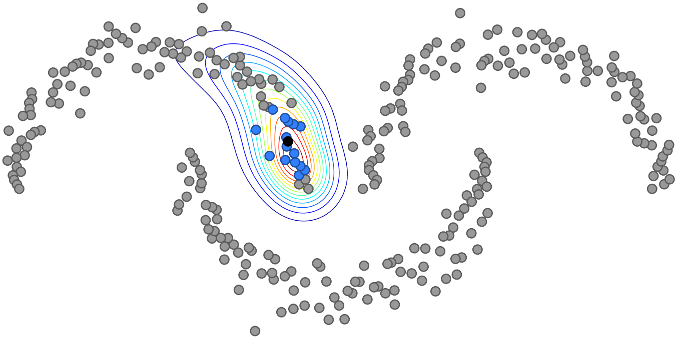
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration 0×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

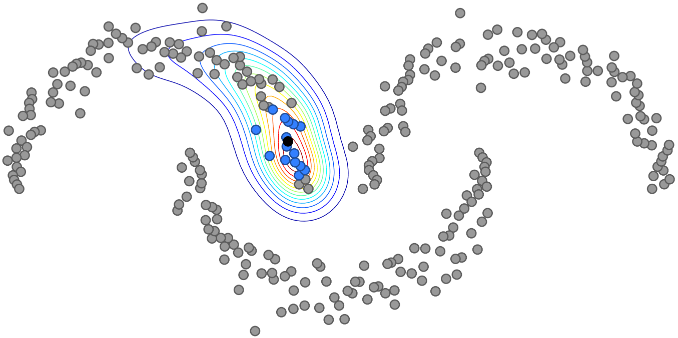
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration 1×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

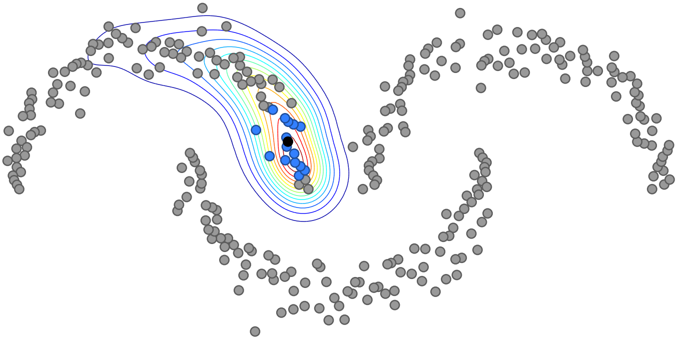
ranking on manifolds: single query



- data points (•), query point (•), nearest neighbors (•)
- iteration 2×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

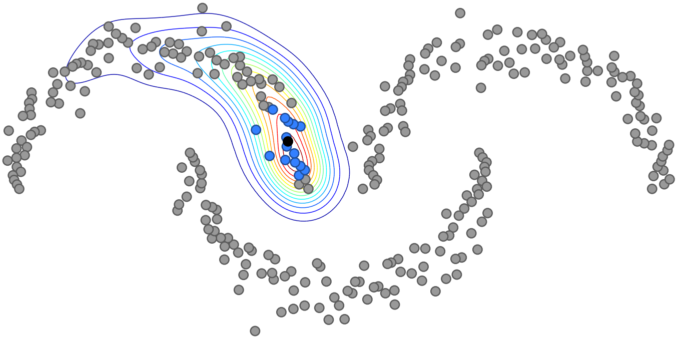
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration $3 \times {\color{orange}30}$

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

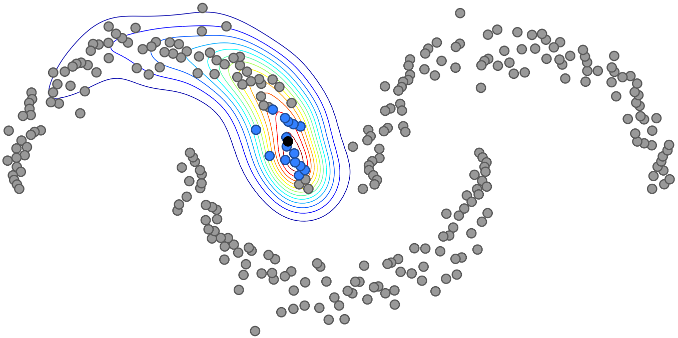
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration 4×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

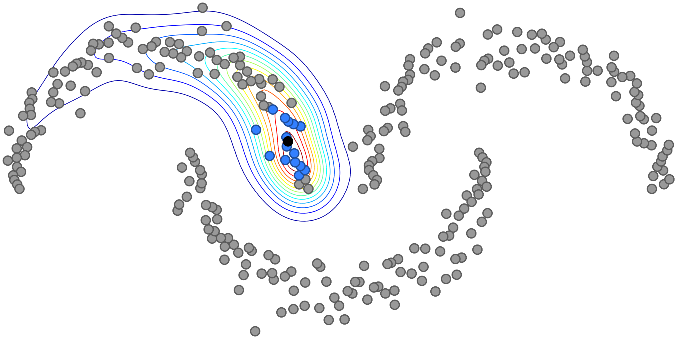
ranking on manifolds: single query



- data points (•), query point (•), nearest neighbors (•)
- iteration 5×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

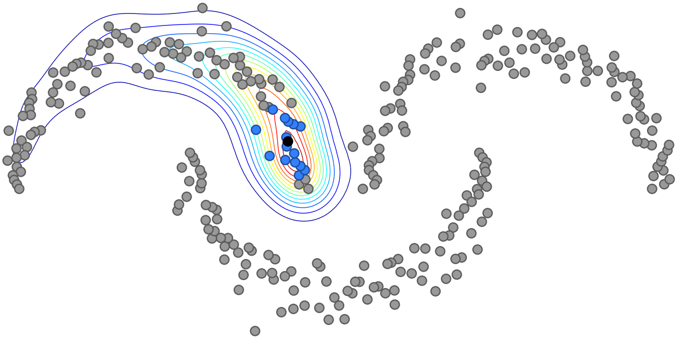
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration 6×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

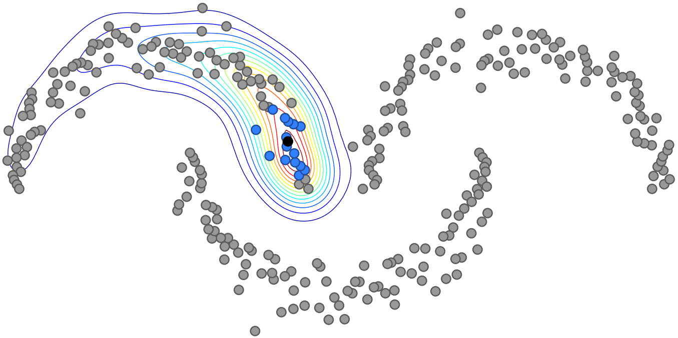
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration $7 \times \color{orange}{30}$

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

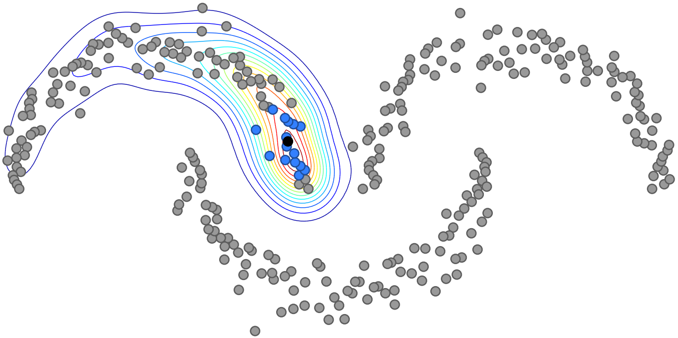
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (∘)
- iteration 8×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

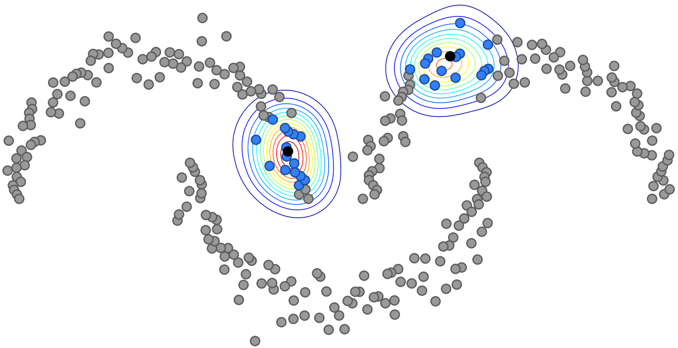
ranking on manifolds: single query



- data points (∘), query point (•), nearest neighbors (•)
- iteration 9×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

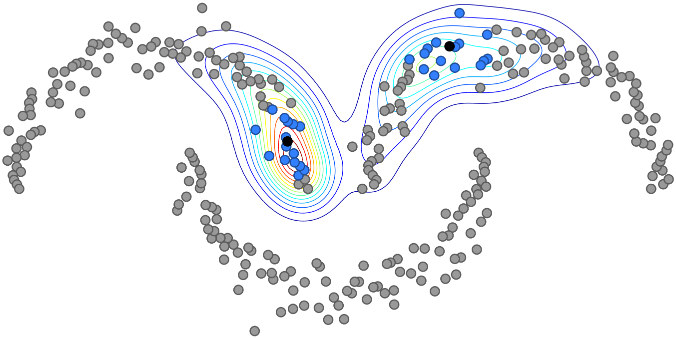
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 0×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

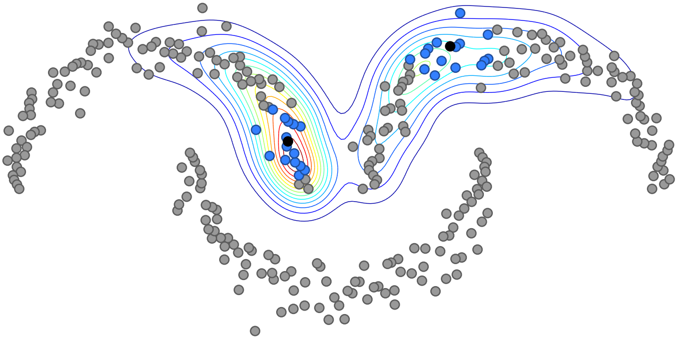
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 1×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

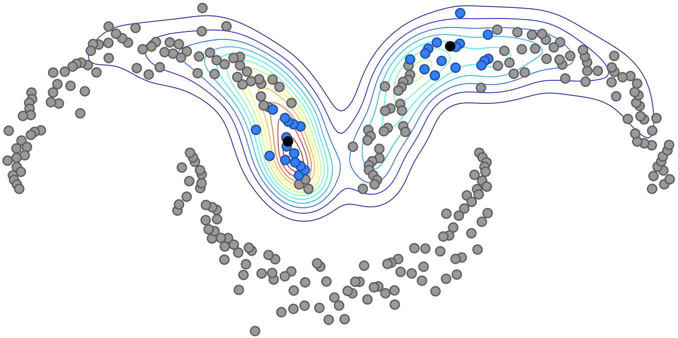
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 2×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

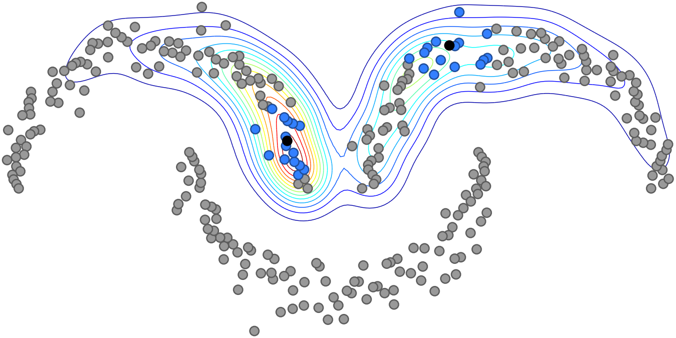
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 3×30

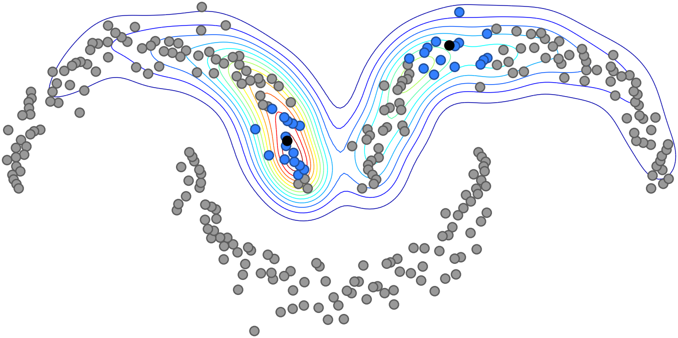
Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 4×30

ranking on manifolds: multiple queries



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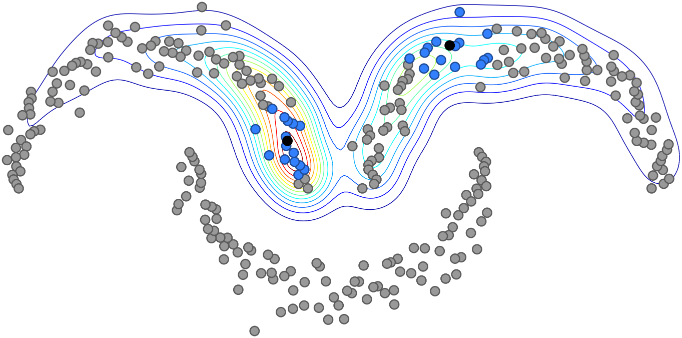
- iteration 5×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

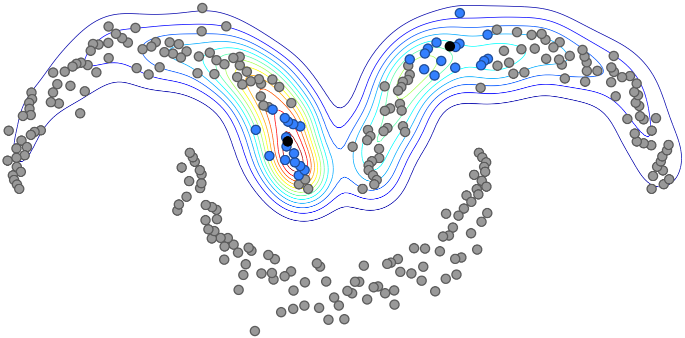
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration $6 \times \color{orange}{30}$

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

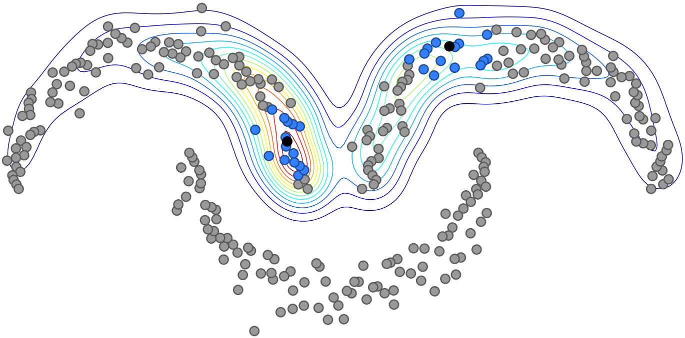
ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 7×30

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

ranking on manifolds: multiple queries



- data points (∘), query points (•), nearest neighbors (∘)
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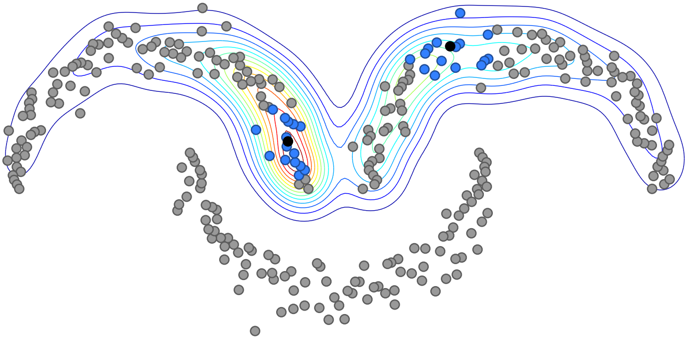

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ranking on manifolds: random walk

[Zhou et al. 2003]

- reciprocal k-nearest neighbor graph on n data points
- non-negative, symmetric, sparse adjacency matrix $W \in \mathbb{R}^{n \times n}$, with zero diagonal (no self-loops)
- symmetrically normalized adjacency matrix

$$\mathcal{W} := D^{-1/2} W D^{-1/2}$$

  where $D = \operatorname{diag}(W\mathbf{1})$ is the degree matrix
- query: vector $\mathbf{y} \in \mathbb{R}^n$ with $y_i = \mathbb{1}[i \text{ is query}]$
- random walk: starting with any $\mathbf{f}^{(0)} \in \mathbb{R}^n$, iterate

$$\mathbf{f}^{(\tau)} = \alpha \mathcal{W} \mathbf{f}^{(\tau-1)} + (1-\alpha)\mathbf{y}$$

  where $\alpha \in [0, 1)$ (typically close to 1)
- rank data points by descending order of $\mathbf{f}$

Zhou, Weston, Gretton, Bousquet and Schölkopf. NIPS 2003. Ranking on Data Manifolds.

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ranking as solving a linear system

[Iscen et al. 2017]

- query: sparse vector $\mathbf{y} \in \mathbb{R}^n$ with nearest neighbor similarities

$$y_i = \sum_{\mathbf{q} \in Q} s(\mathbf{q}, \mathbf{x}_i) \times \mathbb{1}[\mathbf{x}_i \in \mathrm{NN}_X^k(\mathbf{q})]$$

where $Q, X \subset \mathbb{R}^d$ query/data points, $\mathbf{x}_i \in X$, s similarity function

- regularized Laplacian

$$\mathcal{L}_\alpha = \frac{I - \alpha \mathcal{W}}{1 - \alpha}$$

- solve linear system

$$\mathcal{L}_\alpha \mathbf{f} = \mathbf{y}$$

by conjugate gradient method

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

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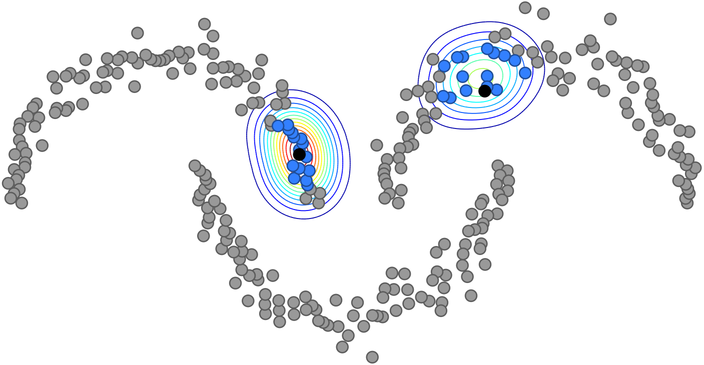
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (∘)
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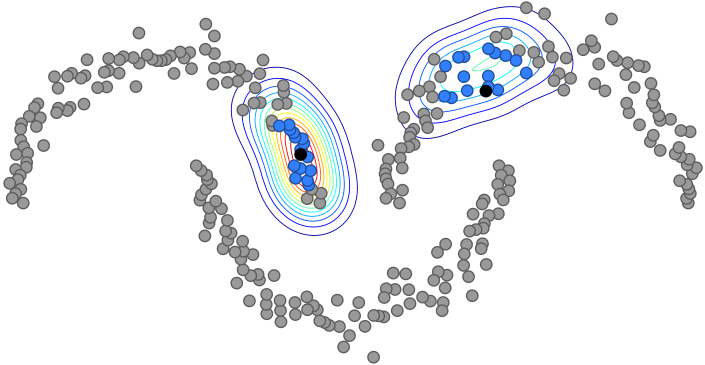
Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

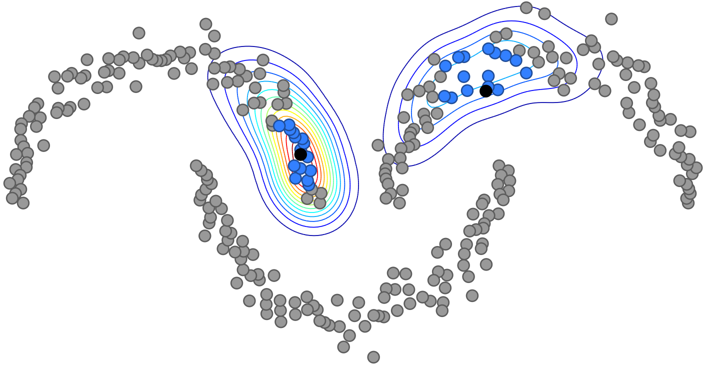
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (•)
- iteration 1×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

ranking by conjugate gradient



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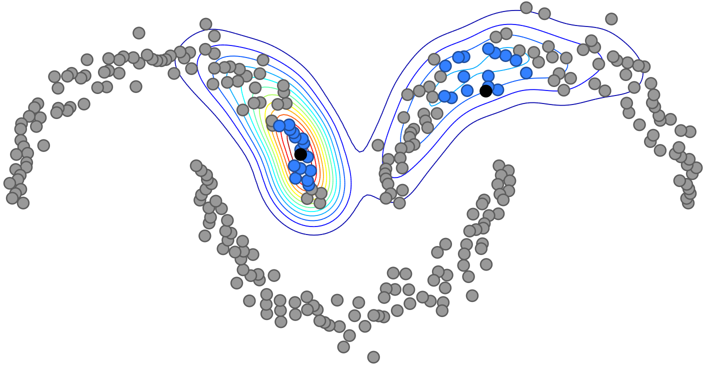
- iteration 2×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

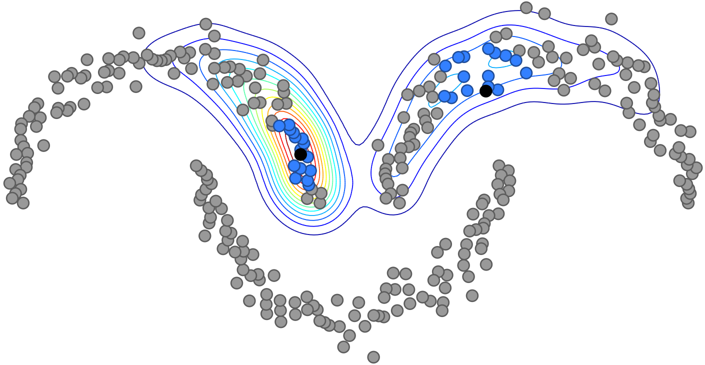
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 3×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

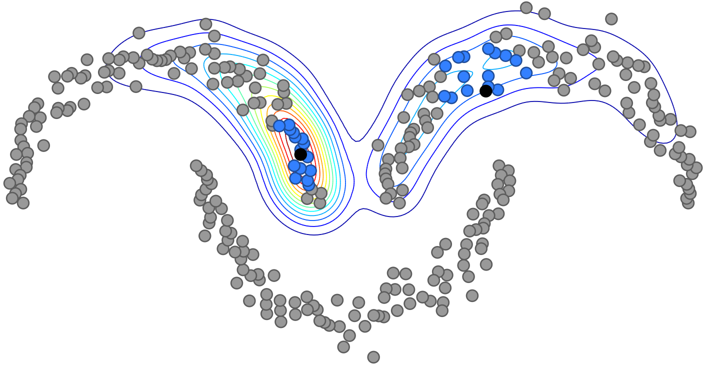
ranking by conjugate gradient



- data points (•), query points (•), nearest neighbors (•)
- iteration 4×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

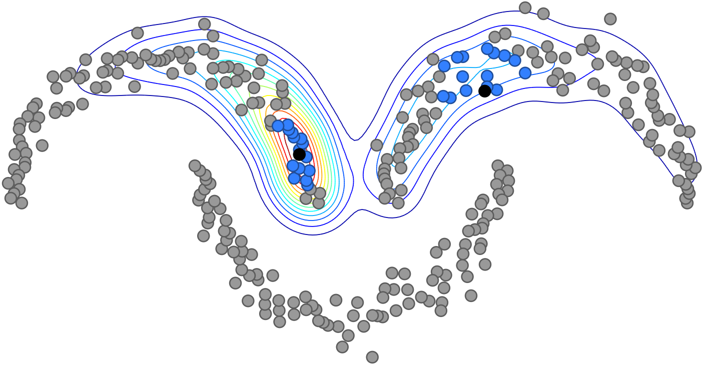
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (•)
- iteration 5×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

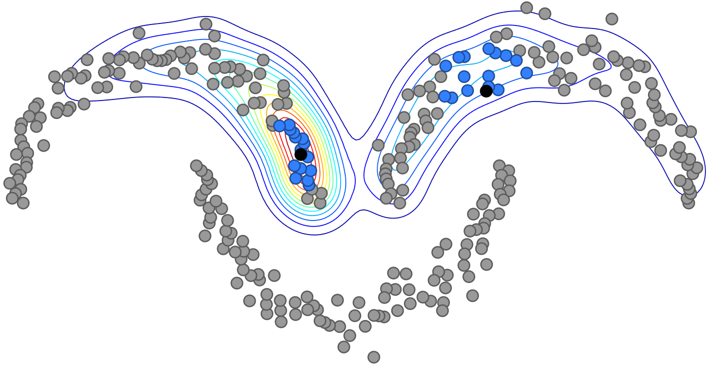
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 6×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

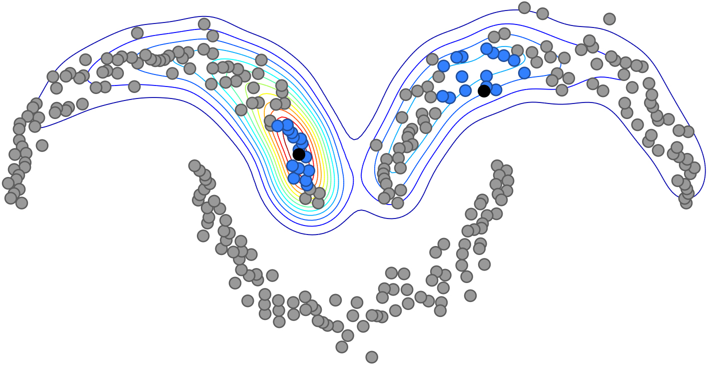
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (•)
- iteration 7×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

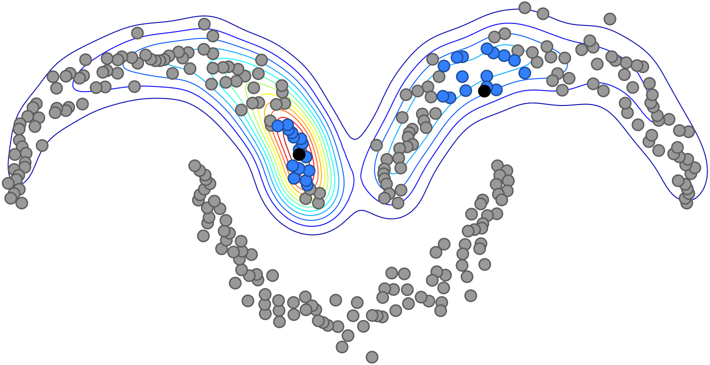
ranking by conjugate gradient



- data points (∘), query points (•), nearest neighbors (∘)
- iteration 8×2

Iscen, Tolias, Avrithis, Furon and Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

ranking by conjugate gradient



- data points (•), query points (•), nearest neighbors (•)
- iteration 9×2

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ranking as solving a linear system

- represent image by global descriptor or multiple regional descriptors
- perform initial query based on Euclidean nearest neighbors
- re-rank by solving linear system
- ResNet-101 fine-tuned by BoW + R-MAC + re-ranking:
    - mAP 87.1 (95.8) on Oxford5k, 96.5 (96.9) on Paris6k
    - 1 (21) descriptors/image $\times$ 2048 dimensions

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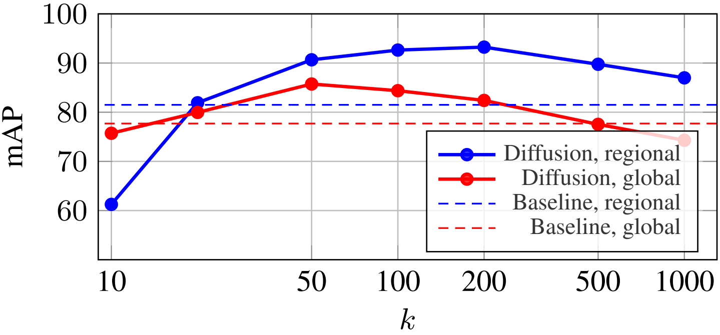
dependence on neighbors, k (Oxford5k)



"small patterns appear more frequently than entire images"

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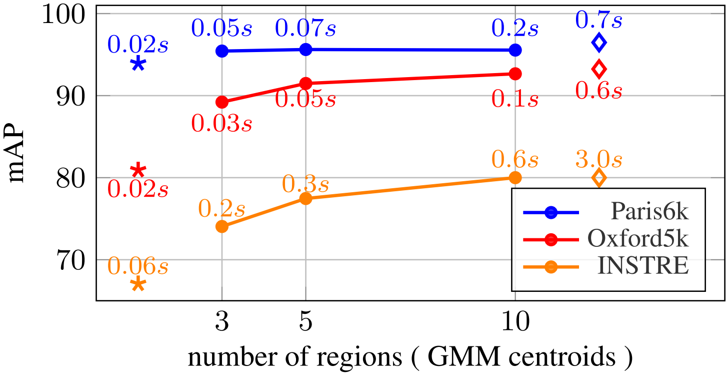
global → regional



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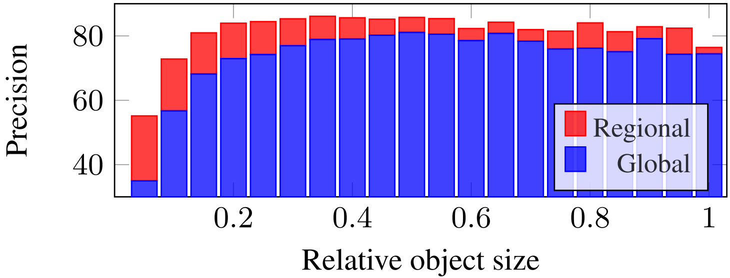
small objects (INSTRE)



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fast spectral ranking

faster than CG?

- want to solve $\mathcal{L}_\alpha \mathbf{x} = \mathbf{y}$
- could invert $\mathcal{L}_\alpha$ offline, but it wouldn't be sparse
- could approximate $\mathcal{L}_\alpha^{-1}$ by $\Phi^\top \Phi$ where Φ is a (sparse) $r \times n$ matrix with $r \ll n$; then

$$\mathbf{x} \approx \Phi^\top \Phi \mathbf{y}$$

- but how to compute Φ without ever inverting $\mathcal{L}_\alpha$?
- still, there is no generalization; even α is given in advance

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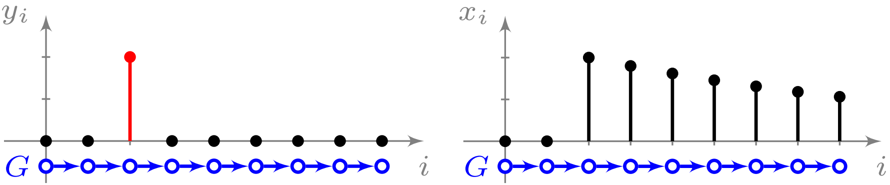
searching on manifolds as smoothing

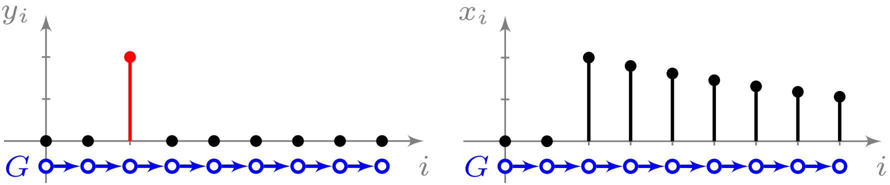
[Iscen et al. 2018]



- exponential moving average filter
- output given by $x_i := (1-\alpha)\sum_{t=0}^{\infty} \alpha^t y_{i-t}$
- or by recurrence $x_i = \alpha x_{i-1} + (1-\alpha) y_i$
- impulse response $h_i = (1-\alpha)\alpha^i u_i$
- transfer function $H(z) := (1-\alpha)\sum_{t=0}^{\infty}(az^{-1})^t = (1-\alpha)/(1-\alpha z^{-1})$

searching on manifolds as smoothing

[Iscen et al. 2018]



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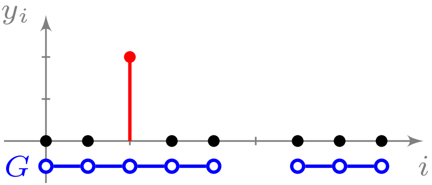
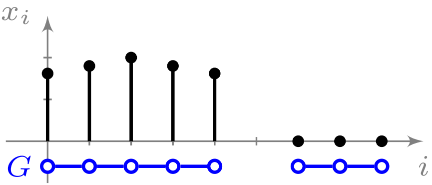
Iscen, Avrithis, Tolias, Furon, Chum. CVPR 2018. Fast Spectral Ranking for Similarity Search.

searching on manifolds as smoothing

[Iscen et al. 2018]





- using a weighted undirected graph G instead
- information "flows" in all directions, controlled by edge weights

Iscen, Avrithis, Tolias, Furon, Chum. CVPR 2018. Fast Spectral Ranking for Similarity Search.

searching on manifolds as smoothing

- express $\mathcal{L}_\alpha^{-1}$ using a transfer function

$$\mathcal{L}_\alpha^{-1} = h_\alpha(\mathcal{W}) = (1-\alpha)(I - \alpha\mathcal{W})^{-1}$$

- given any matrix function h, we want to compute

$$\mathbf{x} = h(\mathcal{W})\mathbf{y}$$

without computing $h(\mathcal{W})$

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searching on manifolds as smoothing

$$\mathbf{x} = h\left(\mathcal{W}\right)\mathbf{y}$$

- eigenvalue decomposition of $\mathcal{W}$
- low-rank approximation
- (under conditions on h and Λ)
- dot-product search
- linear graph filter in frequency domain

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searching on manifolds as smoothing

$$\mathbf{x} = h\left(U \; \Lambda \; U^\top \right) \mathbf{y}$$

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$$\mathbf{x} \approx h\left(U \; \Lambda \; U^\top \right) \mathbf{y}$$

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searching on manifolds as smoothing

$$\mathbf{x} \approx U \, h\left(\Lambda \right) U^\top \mathbf{y}$$

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searching on manifolds as smoothing

$$\mathbf{x} \approx U \, h\Big(\underbrace{\Lambda}_{\text{diagonal}}\Big) \, U^\top \underbrace{\mathbf{y}}_{\text{sparse}}$$

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searching on manifolds as smoothing

$$\boxed{\mathbf{x}} \approx \boxed{\mathcal{F}^{-1}} \; h \left(\boxed{\Lambda} \right) \boxed{\mathcal{F}} \boxed{\mathbf{y}}$$

- eigenvalue decomposition of $\mathcal{W}$
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interpretation: graph signal processing

- in discrete signal processing, a signal of period n is a vector $\mathbf{s} \in \mathbb{R}^n$ where $s_{\bar{i}} := s_{(i \bmod n)+1}$ for $i \in 1, \ldots, n$
- a shift of $\mathbf{s}$ is the mapping $s_{\bar{i}} \mapsto s_{\overline{i-1}}$; also represented by $\mathbf{s} \mapsto C_n \mathbf{s}$ where C_n is an $n \times n$ circulant zero-one matrix, *e.g.* for $n = 5$

$$C_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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interpretation: graph signal processing

- a linear, shift invariant, causal filter is the mapping $\mathbf{s} \mapsto H\mathbf{s}$ where

$$H := h(C_n) = \sum_{t=0}^{\infty} h_t C_n^t$$

- matrix C_n has the eigenvalue decomposition $U \Lambda U^\top$ where $U^\top$ is the $n \times n$ discrete Fourier transform matrix $\mathcal{F}$
- if the series $h(C_n)$ converges, filtering $\mathbf{s} \mapsto H\mathbf{s}$ is written as

$$\mathbf{s} \mapsto \mathcal{F}^{-1} h(\Lambda) \mathcal{F} \mathbf{s}$$

- graph signal processing generalizes the above by replacing C_n with a matrix determined by a graph

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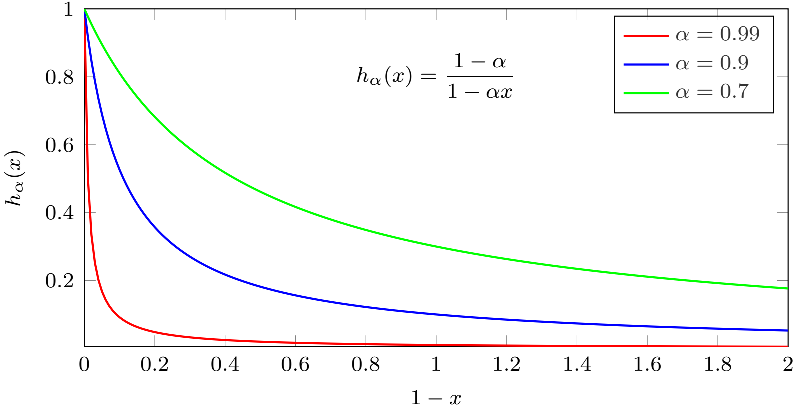

- low-pass filtering in the frequency domain
- or, “soft” dimensionality reduction

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interpretation: random fields

- a Gaussian Markov random field (GMRF) with precision A and mean $\boldsymbol{\mu}$ can be parametrized as

$$p(\mathbf{x}) := \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, A^{-1}) \propto e^{-E(\mathbf{x}|\mathbf{b},A)}$$

  where $E(\mathbf{x}|\mathbf{b}, A) := \frac{1}{2}\mathbf{x}^\top A\mathbf{x} - \mathbf{b}^\top\mathbf{x}$ is a quadratic energy
- its expectation $\boldsymbol{\mu} = A^{-1}\mathbf{b}$ is the minimizer of this energy
- our solution $\mathbf{x}^* = \mathcal{L}_\alpha^{-1}\mathbf{y}$ is the expectation of a GMRF with energy

$$f_\alpha(\mathbf{x}) := E(\mathbf{x}|\mathbf{y}, \mathcal{L}_\alpha) = \frac{1}{2}\mathbf{x}^\top \mathcal{L}_\alpha \mathbf{x} - \mathbf{y}^\top\mathbf{x}$$

- if $\hat{\mathbf{x}} := D^{-1/2}\mathbf{x}$, this energy has the same minimizer as

$$\alpha \sum_{i,j} w_{ij} \left\|\hat{x}_i - \hat{x}_j\right\|^2 + (1-\alpha)\left\|\mathbf{x} - \mathbf{y}\right\|^2$$

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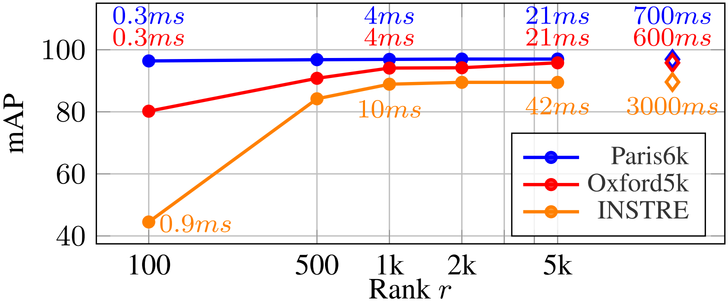
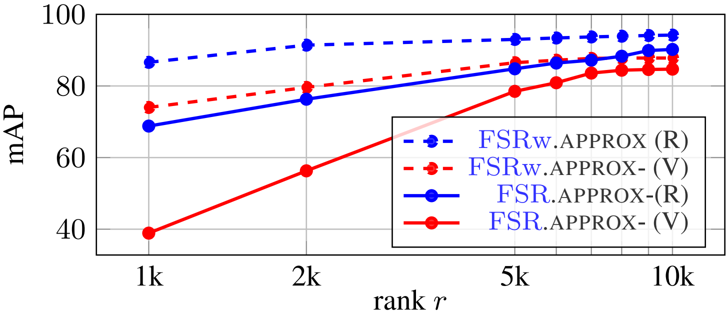
small scale (21 descriptors/image)



- in summary: same performance as CG, two orders of magnitude faster, but $3\times$ space needed

Oxford105k (5 descriptors/image)



- in summary: same performance as CG, two orders of magnitude faster, but $3\times$ space needed

hard examples?

- red: drift
- blue: incorrect annotations

mining on manifolds

embeddings for manifold similarity

- the $n \times n$ kernel matrix

$$K = h(\mathcal{W}) = U h(\Lambda) U^\top$$

expresses the pairwise manifold similarity of all data points

- if $h(x) > 0$ for $x \in \mathbb{R}$, which holds for h_α, then K is positive-definite and there is an $n \times n$ matrix Φ such that $K = \Phi^\top \Phi$
- a particular choice is

$$\Phi = h(\Lambda)^{1/2} U^\top$$

- if we choose a rank-r approximation instead, then Φ is $r \times n$ and defines a low-dimensional embedding onto $\mathbb{R}^r$
- so why not learn an embedding on a training set such that it generalizes manifold similarity to other data sets?

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mining on manifolds

[Iscen et al. 2018]

- data points (∘), query point $\mathbf{x}$ (•)

Iscen, Tolias, Avrithis and Chum. CVPR 2018. Mining on Manifolds: Metric Learning without Labels.

mining on manifolds

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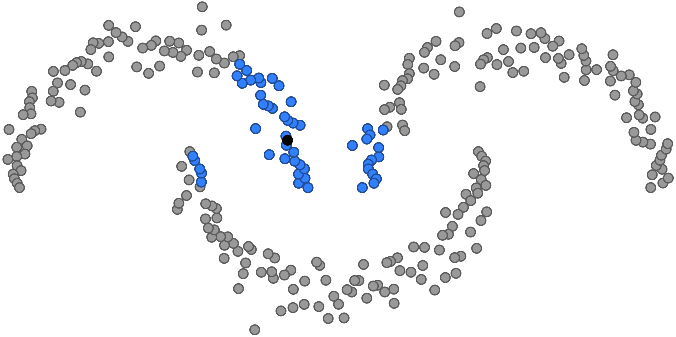

- data points (∘), query point $\mathbf{x}$ (•)
- Euclidean nearest neighbors $E(\mathbf{x})$ (∘)

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mining on manifolds

[Iscen et al. 2018]

- data points (∘), query point $\mathbf{x}$ (•)
- manifold nearest neighbors $M(\mathbf{x})$ (∘)

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mining on manifolds

[Iscen et al. 2018]

- data points (∘), query point $\mathbf{x}$ (•)
- hard positives $S^+ = M(\mathbf{x}) \setminus E(\mathbf{x})$ (•)

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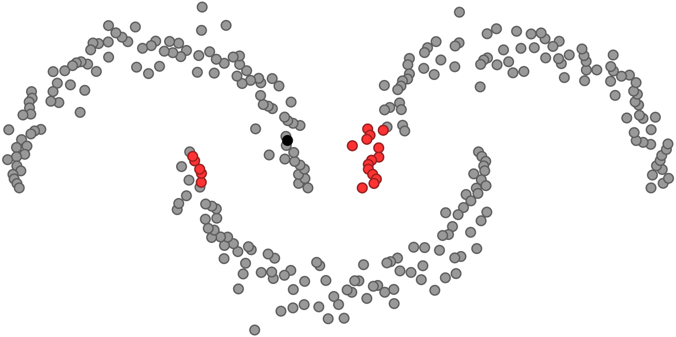
mining on manifolds

[Iscen et al. 2018]



- data points (∘), query point $\mathbf{x}$ (•)
- hard negatives $S^- = E(\mathbf{x}) \setminus M(\mathbf{x})$ (•)

Iscen, Tolias, Avrithis and Chum. CVPR 2018. Mining on Manifolds: Metric Learning without Labels.

hard positive/negative examples

- query (anchor) $(\mathbf{x})$
- positives $S^+(\mathbf{x})$ *vs.* Euclidean neighbors $E(\mathbf{x})$
- negatives $S^-(\mathbf{x})$ *vs.* Euclidean non-neighbors $X \setminus E(\mathbf{x})$

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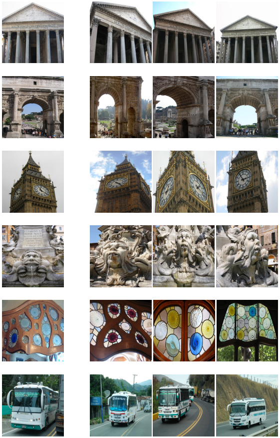

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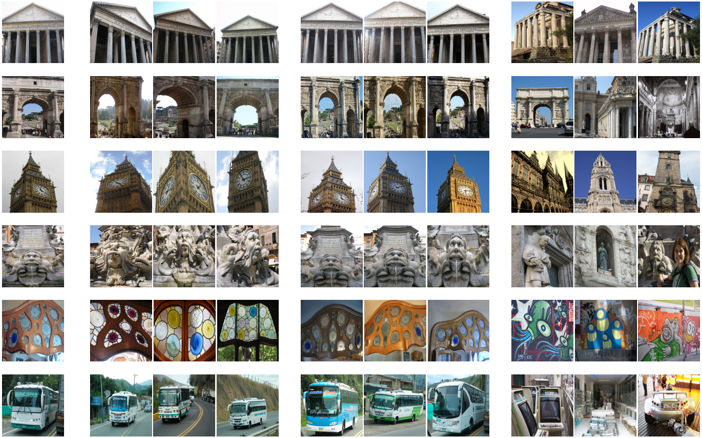

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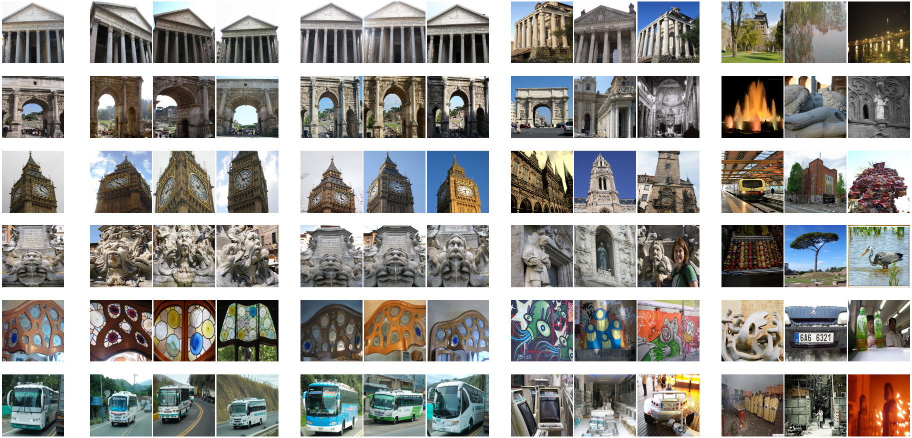

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fine-tuning with hard example mining

- pre-train network
- extract descriptors on unlabeled dataset
- construct nearest neighbor graph
- sample anchors, measure Euclidean and manifold distances
- sample positives and negatives
- fine-tune using contrastive or triplet loss

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fine-grained categorization results

Method	Labels	R@1	R@2	R@4	R@8	NMI
Initial	No	35.0	46.8	59.3	72.0	48.1
Triplet+semi-hard	Yes	42.3	55.0	66.4	77.2	55.4
Lifted-Structure	Yes	43.6	56.6	68.6	79.6	56.5
Triplet+	Yes	45.9	57.7	69.6	79.8	58.1
Clustering	Yes	48.2	61.4	71.8	81.9	59.2
Triplet+++	Yes	49.8	62.3	74.1	83.3	59.9
Cyclic match	No	40.8	52.8	65.1	76.0	52.6
Ours	No	45.3	57.8	68.6	78.4	55.0

- CUB200-2011 dataset, 200 bird species, 100 training / 100 testing
- GoogLeNet pre-trained on ImageNet, then fine-tuned with triplet loss

particular object retrieval results

Model	Pooling	Labels	Oxf5k	Oxf105k	Par6k	Par106k	Hol	Instre
ImageNet	MAC	Human	58.5	50.3	73.0	59.0	79.4	48.5
From BoW		SfM	**79.7**	73.9	82.4	74.6	81.4	48.5
Ours		—	78.7	**74.3**	**83.1**	**75.6**	**82.6**	**55.5**
ImageNet	R-MAC	Human	68.0	61.0	76.6	72.1	87.0	55.6
From BoW		SfM	77.8	70.1	84.1	76.8	84.4	47.7
Ours		—	**78.2**	**72.6**	**85.1**	**78.0**	**87.5**	**57.7**

- VGG-16 pre-trained on ImageNet, then fine-tuned with constrastive loss on a 1M unlabeled dataset with MAC representation
- at test time, either MAC or R-MAC used

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summary

- pooling CNN representations is best at last convolutional layers
- fine-tuning with constrastive or triplet loss allows transferring to a new domain and learning to rank as opposed to classify
- now that images are represented by a global descriptor or just a few regional descriptors, graph methods are more applicable than ever
- it turns out that query expansion is not just "post processing" but at the core of ranking on manifolds
- there is at least one low-dimensional embedding of manifold similarity, but is dataset-specific
- modeling the manifold explicitly allows unsupervised fine-tuning without labels, auxiliary systems (*e.g.* SIFT pipeline), or other information (*e.g.* temporal neighborhood in video)

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