## Overview

- Regions of arbitrary scale and shape, without scale space construction
- Weighted distance map on image gradient, using our exact, linear-time algorithm. - Weighted medial axis by generalizing chord residue, typically used in Voronoi skeletons.
- Decompose medial axis into a graph representing image structure in terms of peaks and saddle points.
- Reconstruct regions using the same propagation method, due to a duality property.

- Join regions and select those that are well enclosed by boundaries, in agreement with the

Gestalt principle of closure; incomplete boundaries allowed
$\bullet$ Select whole regions and their parts independently.

- Binary code and documentation at http://image.ntua.gr/iva/tools/mfd/


## Examples



## Weighted distance transform

- Given input $f$ in domain $X$, define its weighted distance map $h=\mathcal{D}(f)$ as

$$
h(x)=\bigwedge_{y \in X} d(x, y)+f(y), \quad x \in X
$$

where $\wedge$ denotes infimum and $d$ Euclidean distance.

- Given image $f_{0}$, compute gradient magnitude $g=\left|\nabla f_{0}\right|$ and define $f(x)=\sigma / g(x)$ for $x \in X$, where $\sigma$ is a scale parameter-generalizing the $0 / \infty$ indicator function.
- Exact group marching (EGM) algorithm: move all points on the propagating front as a group, in arbitrary order; exact result, linear in the number of points.

- For each $x \in X$, its source set $S(x)$ is the set of points $y \in X$ for which $d(x, y)+f(y)$ is minimized, and is used to define the medial axis.
- The source set $S(f)$ of $f$ contains all source points of $f$ in $X ; \mathcal{D}(f)$ is uniquely determined by $f$ restricted on $S(f)$.


## Weighted medial axis

$\bullet$ Point $x \in X$ is a medial point if it has at least two distinct sources. The (weighted) medial axis $A(f)$ contains all such points: $A(f)=\{x \in X:|S(x)|>1\}$.

- Weighted medial axis (WMA) algorithm computes $A(f)$ given $\mathcal{D}(f)$ and $S(f)$,
beginning at peaks of $\mathcal{D}(f)$ and propagating based on residue function $r(x)$, generalizing chord residue by Ogniewicz and Kubler.

- Using a graph to represent the topology of the source set $S(f)$, compute $r(x)$ in constant time for each $x \in X$; hence WMA is linear in the number of points. - Guarantee that $A(f)$ is connected for each component of $X \backslash S(f)$.


## Feature detection

- Decompose $A(f)$ and construct weighted graph $\mathcal{G}(f)$ such that: (a) vertices correspond to peaks of $\mathcal{D}(f)$; (b) edges correspond to local minima along $A(f)$ (saddle points of $\mathcal{D}(f)$ ); (c) edge weights defined as the height at those points.
- Medial axis decomposition (MAD) algorithm constructs $\mathcal{G}(f)$ given $h=\mathcal{D}(f)$ and $A(f)$; equivalent to watershed of $-h$ on $A(f)$, with peaks as markers.

- Partition image: invoke EGM with input $g(x)=-h(x)$ if $x \in A(f)$, and $+\infty$ otherwise, backpropagating from medial to boundaries; equivalent to watershed of $-h$ on $X$.
- Each edge $e$ of $\mathcal{G}(f)$ is generated at saddle point $x(e)$ with boundary gap $w(x(e))$; then component $\kappa$ with area $a(\kappa)$ and incident edge set $E(\kappa)$ has shape fragmentation factor

$$
\phi(\kappa)=\frac{1}{a(\kappa)} \sum_{e \in E(\kappa)} w^{2}(x(e)) .
$$



- Join components in non-increasing order of edge weights and select component $\kappa$ as a feature if $\phi(\kappa)<\tau$ where threshold $\tau>0$ controls selectivity.


## Matching experiment

- Matching across viewpoint, zoom, rotation, light, and blur; measure performance in terms of repeatability and matching score, using SIFT descriptors for all detectors.



## Retrieval experiment

- Larger scale retrieval using bag of words (BoW); ranking by TF-IDF and fast spatial matching (FastSM); measure performance in terms of mean Average Precision (mAP). - Oxford 5 K dataset, comprising 5,062 images with 55 queries. 200K vocabulary from the same dataset, constructed separately for each detector.

| Detector | Features <br> $\left(\times 10^{6}\right)$ |  | Space |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $(\mathrm{MB})$ | Query (ms) |  | Index |  | Rerank | Index | Rerank

- Indexing space and query time depend on the average number of features per image; the objective is highest mAP with reasonable space/time requirements


## Current and future work

- Exploit exact region shape; other types of features; GPU implementation - Generic image segmentation; edge detection and grouping.
- Shape-based object detection.

