Planar Shape Decomposition Made Simple

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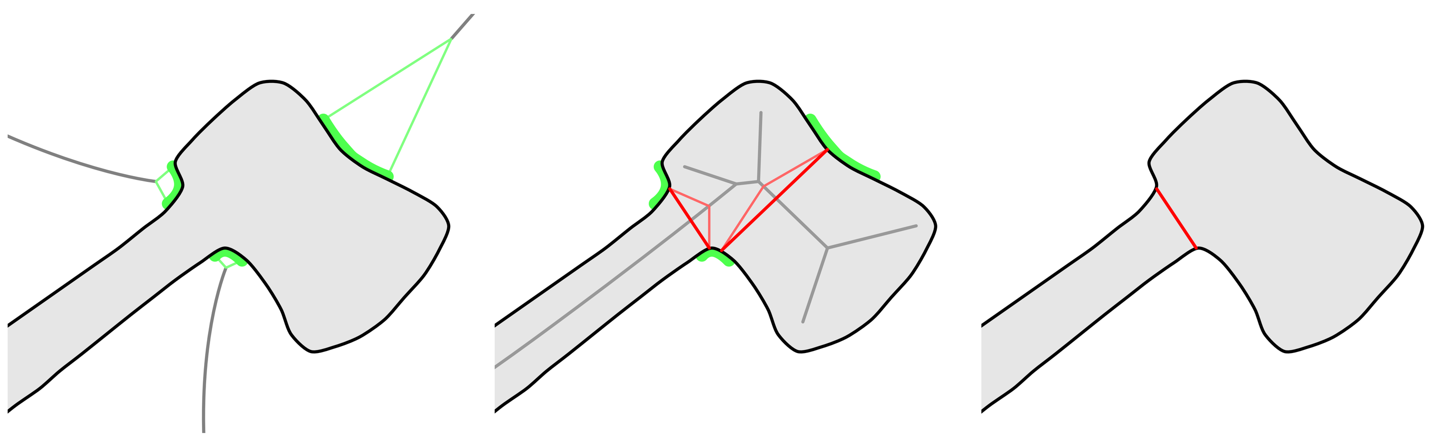
Motivation

- Planar shape decomposition without global optimization or differentiation
- All information available from (exterior and interior) medial axis representation
- Most rules and salience measure from psychophysical studies accommodated in a simple computational model



exterior interior cuts

Shape Representation

- A planar shape is a set $X \subset \mathbb{R}^2$; its boundary ∂X is a finite union of mutually disjoint simple closed curves

Medial axis [1]

- The distance map $\mathcal{D}(X) : X \to \mathbb{R}$ is a function mapping each point $y \in X$ to

$$\mathcal{D}(X)(y) = \inf_{x \in \partial X} \|y - x\|$$

- For $y \in \mathbb{R}^2$, the projection or contact set

$$\pi(y) = \{x \in \partial X : \|y - x\| = \mathcal{D}(X)(x)\}$$

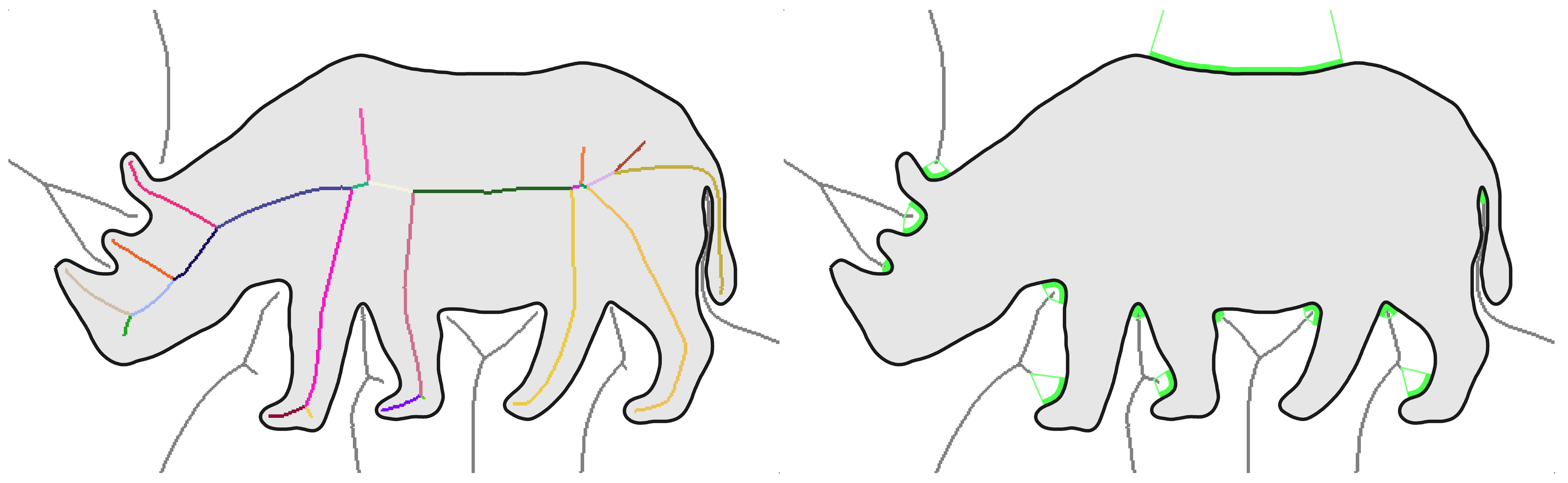
  is the set of points on the boundary at minimal distance to y; each $x \in \pi(y)$ is a projection or contact point of y
- The (interior) medial axis

$$\mathcal{M}(X) = \{x \in \mathbb{R}^2 : |\pi(x)| > 1\}$$

  is the set of points with more than one projection points
- The exterior medial axis of X is the medial axis of its complement $\mathbb{R}^2 \setminus X$

Construction [2]

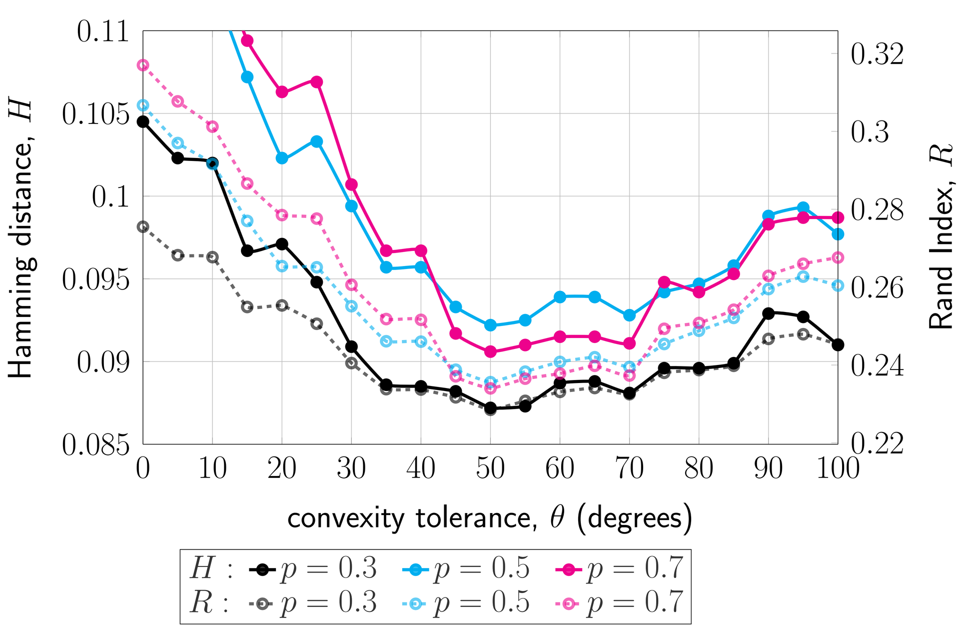
- Given two points $x, y \in \partial X$, the arc length $\ell(x, y)$ is the length of the minimal arc of ∂X having x, y as endpoints or ∞ if no such arc exists
- Given a point z, its chord residue $r(z) = \sup_{x,y \in \pi(z)} \ell(x, y) - \|x - y\|$ is the maximal difference between arc length and chord length over all pairs of points in its projection
- Construction begins at local maxima of distance map and propagates as long as the residue is higher than a given threshold $\sigma > 0$



exterior & interior medial axis concave corners

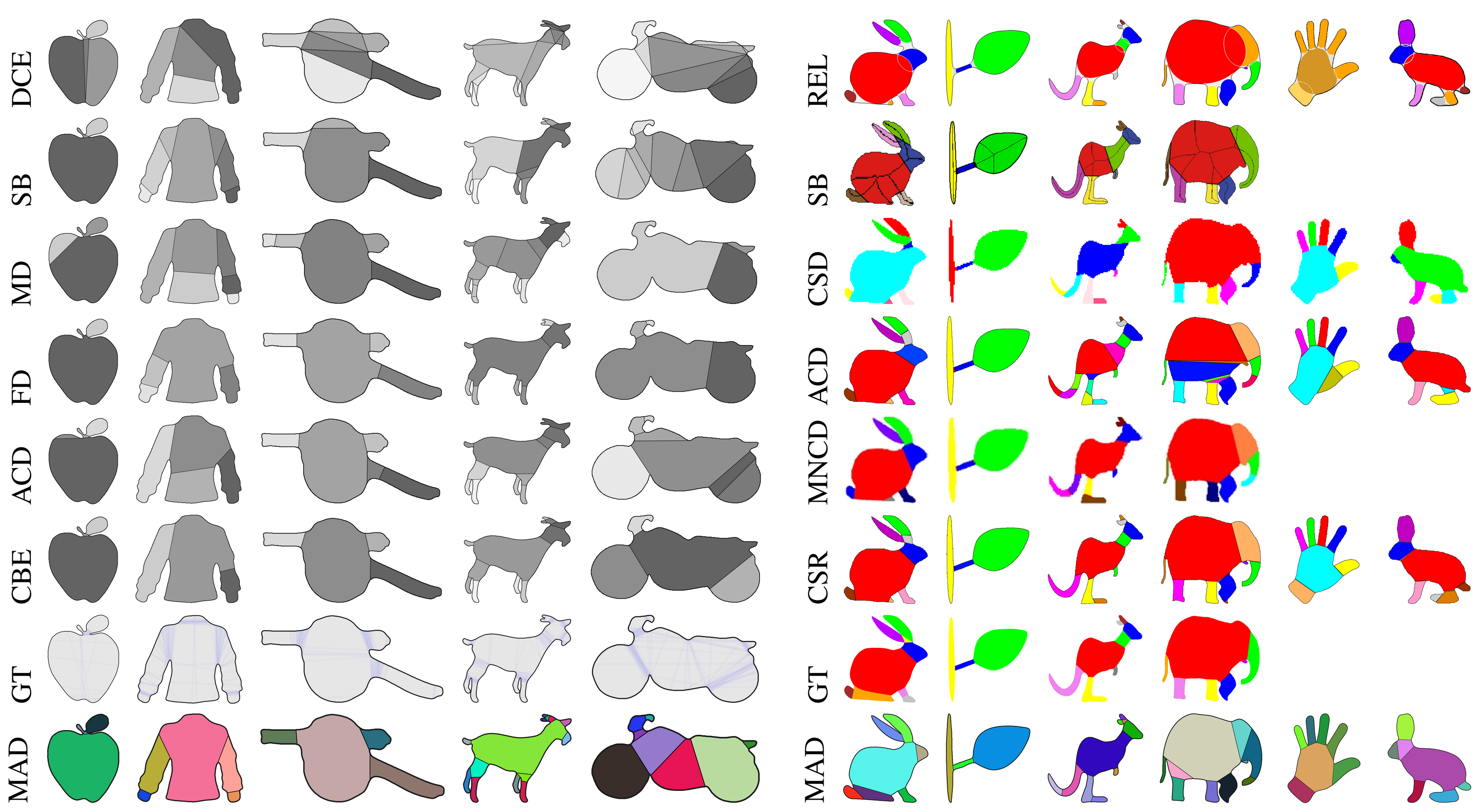
Quantitative Evaluation

- Evaluation measures: *Hamming distance* and *Rand Index (Jaccard distance)*



	average		majority	
	H	R	H	R
DCE	0.208	0.497	0.188	0.466
SB	0.163	0.402	0.131	0.335
MD	0.151	0.371	0.126	0.328
FD	0.145	0.350	0.112	0.267
ACD	0.128	0.323	**0.092**	0.251
MAD	**0.126**	**0.317**	0.096	**0.247**
MAD-opt	0.118	0.303	0.085	0.225
CBE	**0.111**	**0.288**	**0.069**	**0.186**
Human	0.128	0.312	0.093	0.245
Baseline	0.160	0.424	0.140	0.376

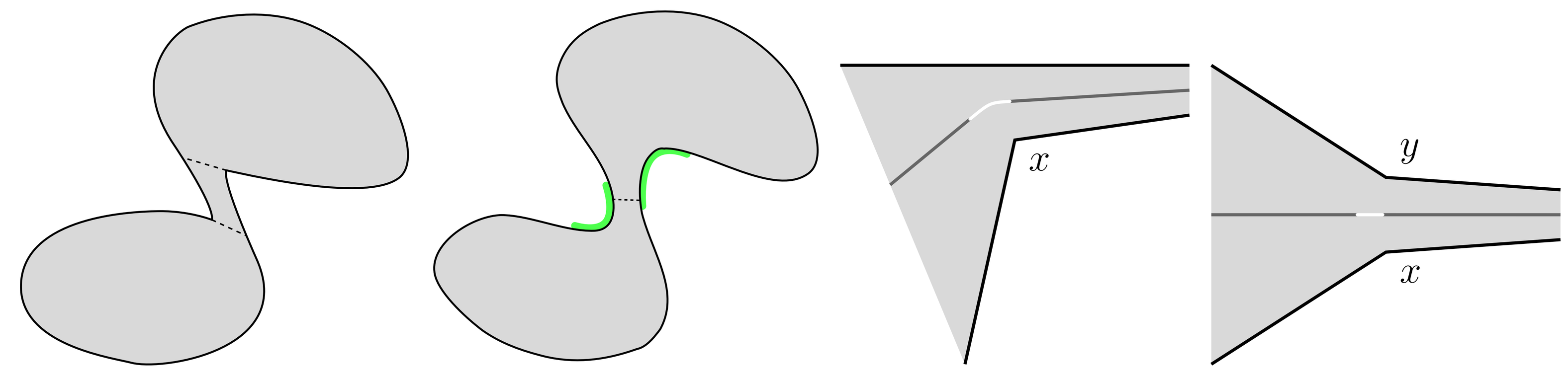
Qualitative Evaluation



Shape Decomposition

Minima rule

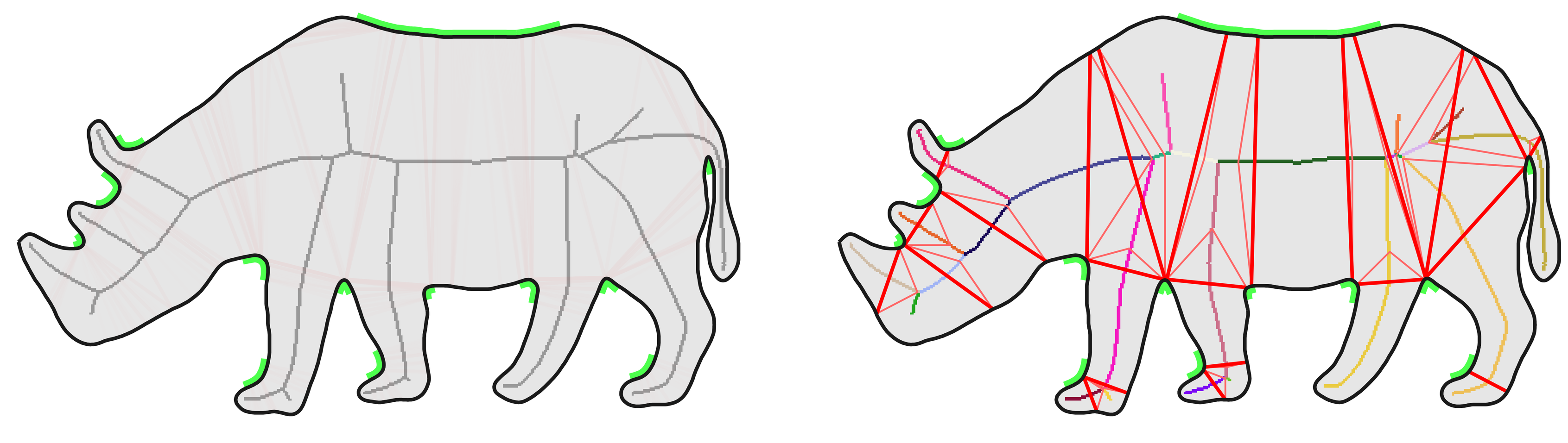
- A shape should be cut at points of negative minima of curvature [3]
- But these are exactly projection points of end vertices of the exterior medial axis [1]
- Moreover, one may get not just one boundary point but an entire arc, called a (concave) corner
- Without differentiation, an end-vertex with its two projection points determine the position, spatial extent, orientation and strength of each concavity



sharp corners weak corners semi-ligature full ligature

Symmetry

- A cut of a shape X is a line segment connecting two points of ∂X
- All prior work examines all possible pairs of points on ∂X as candidate cut endpoints; we only consider pairs of points that are projection points of the same point of the interior medial axis
- A cut may have one or two corner points as endpoints, called single or double cut respectively
- Raw cuts: traverse interior medial axis collecting all pairs of projection points such that at least one lies on a corner; this is stronger than requiring cuts to cross an axis of local symmetry [3]



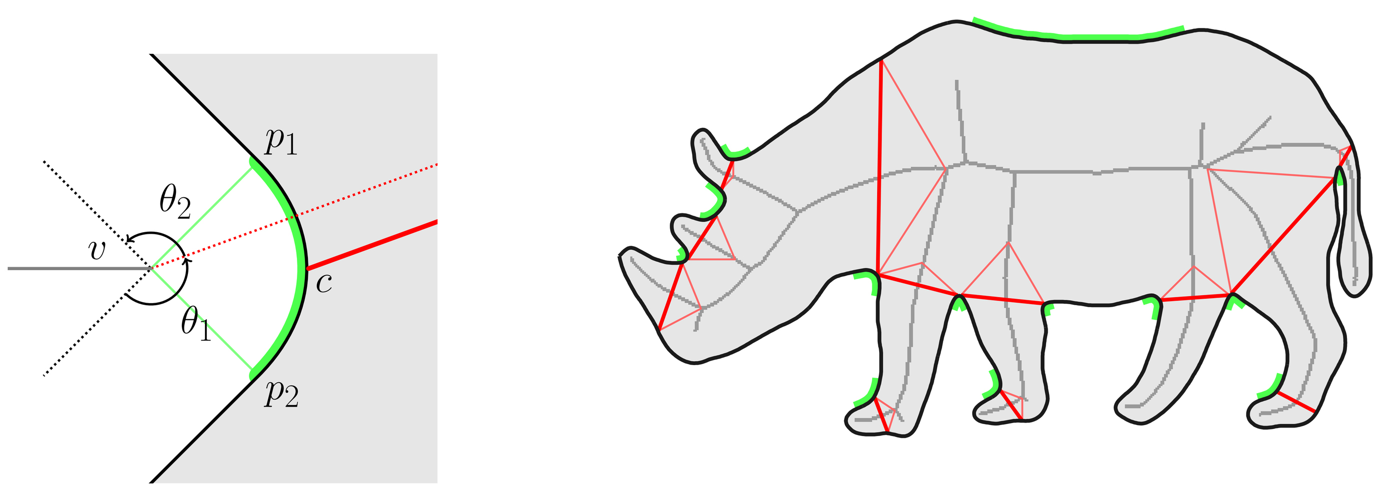
raw cuts candidate cuts

Equivalence

- Select candidate cuts by applying equivalence rules on raw cuts
- Branch equivalence: two cuts on the same branch whose endpoints share at least one corner; double cuts have priority over single cuts
- Corner equivalence: two (double) cuts whose endpoints lie on the same pair of corners; the cut with the maximal protrusion strength is selected

Salience measures

- Protrusion strength: ratio of cut length to arc length; select cuts with protrusion less than p



local convexity selected cuts

Local convexity & short-cut rule

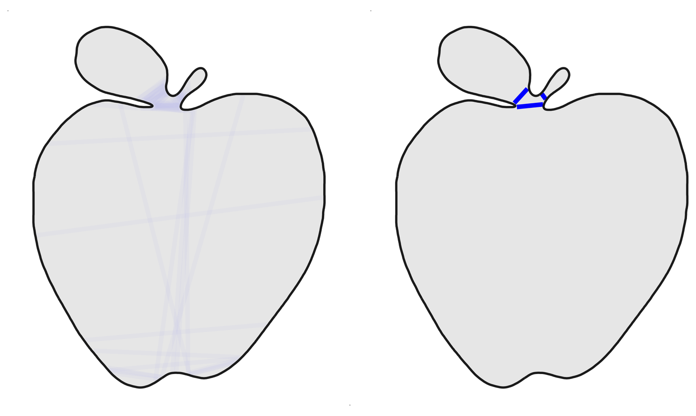
- Most approaches seek the minimal number of cuts such that each shape part is approximately convex
- But negative minima of curvature are exactly points where the shape is locally maximally concave
- For each corner, we select independently the minimal number of cuts such that the interior angle of each part is less than $\pi + \theta$, where θ is a tolerance
- Priority given according to short-cut rule [4], but arbitrary salience measures apply

Dataset

- Snodgrass and Vanderwart (S&V) everyday object dataset contains 260 line drawings
- De Winter and Wagemans dataset [5] evaluates exactly segmentation of 88 object outlines
- The subset has been converted to smooth outlines and each segmented by 39.5 subjects on average
- For each shape there are 122.4 part-cuts, that is 3.1 cuts per subject on average

Majority Voting

- Part-cuts of human subjects are typically inconsistent: evaluate on majority cuts
- Apply agglomerative clustering on all human cuts according to arc distance
- Select cluster representatives by averaging endpoints on the parametrization of the boundary curve
- Discard cluster with less than t votes



all subjects majority

References

[1] Choi *et al.* Mathematical theory of medial axis transform. *Pacific Journal of Mathematics*, 1997.
[2] Avrithis & Rapantzikos. The medial feature detector: stable regions from image boundaries. *ICCV*, 2011.
[3] Hoffman & Richards. Parts of recognition. *Cognition*, 1984.
[4] Singh *et al.* Parsing silhouettes: the short-cut rule. *Perception and Psychophysics*, 1999.
[5] De Winter & Wagemans. Segmentation of object outlines into parts. *Cognition*, 2006.

http://image.ntua.gr/iva/research/cuts Contact: papanelo@image.ntua.gr, iavr@image.ntua.gr