# AN EFFICIENT SCHEME FOR INVARIANT OPTICAL CHARACTER RECOGNITION USING TRIPLE CORRELATIONS 

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## Optical Character Recognition

- Analysis of the document into its constituents (photographs, figures and text).
- Segmentation of the text into columns, paragraphs, lines, words and characters.
- Recognition of the segmented characters.
- Ambiguity resolution.

Also:

- Preprocessing: gray scale normalization, noise elimination.
- Postprocessing: spelling verification or correction, customized lexicons.
- Interaction with human operators.


# Recognition of segmented characters 

## - Feature extraction

Input image $\longrightarrow$ Vector of fixed length, containing all the "essential" characteristics.

- Classification

Class discrimination by generalizing from a training set.

## Goals:

1. Derivation of a feature vector which is:

- Invariant to shift, rotation and scale [SRS] transformations.
- Insensitive to noise and small distortions.

2. Fast implementation in the case of binary images.

## Solution:

- Calculation of the $3^{r d}$ order correlation of the pattern.
- Appropriate clustering \& transformation of the triple-correlation domain.


## Definition \& properties of $3^{r d}$ order correlations

$x(\mathbf{t})$ : real 2-D signal with support

$$
S=\{0, \ldots, N-1\} \times\{0, \ldots, N-1\}
$$

$$
x_{3}\left(\tau_{1}, \tau_{2}\right) \triangleq \frac{1}{N^{2}} \sum_{S} x(\mathbf{t}) x\left(\mathbf{t}+\tau_{1}\right) x\left(\mathbf{t}+\tau_{2}\right), \quad \tau_{1}, \tau_{2} \in S^{\prime}
$$

$$
S^{\prime}=\{-N+1, \ldots, N-1\} \times\{-N+1, \ldots, N-1\}
$$

- Symmetries
- Insensitivity to noise.
- One-to-one correspondence with the original signal.
- Translation, rotation \& scaling:
$y(\mathbf{t})=x\left(\mathbf{T}_{\alpha, \theta} \mathbf{t}+\mathbf{t}_{o}\right) \Longrightarrow$
$y_{3}\left(\tau_{1}, \tau_{2}\right)=x_{3}\left(\mathbf{T}_{\alpha, \theta} \tau_{1}, \mathbf{T}_{\alpha, \theta} \tau_{2}\right)$
where $\boldsymbol{T}_{\alpha, \theta}$ : scaling \& rotation matrix $\mathbf{t}_{o}$ : shifting vector


## The invariant representation

1. Definition of classes:
$C\left(\tau_{1}, \tau_{2}\right)$ : set of all lags which form triangles similar to the one defined by $\tau_{1}, \tau_{2}$
2. Arrangement between members of each class in a log-polar grid:
$\tilde{x}_{3}\left(\rho, \phi ; \tau_{1}, \tau_{2}\right) \triangleq x_{3}\left(\mathbf{T}_{\beta, \phi} \tau_{1}, \mathbf{T}_{\beta, \phi} \tau_{2}\right)$
where $\mathbf{T}_{\beta, \phi}=\beta\left[\begin{array}{cc}\cos \phi & -\sin \phi \\ \sin \phi & \cos \phi\end{array}\right]$ and $\rho=\log \beta$
Then:
$y(\mathbf{t})=x\left(\mathbf{T}_{\alpha, \theta} \mathbf{t}+\mathbf{t}_{o}\right)$
$\tilde{y}_{3}\left(\rho, \phi ; \tau_{1}, \tau_{2}\right)=\tilde{x}_{3}\left(\rho \log \alpha, \phi+\theta ; \tau_{1}, \tau_{2}\right)$
3. Calculation of the 2-D Fourier transform $\tilde{X}_{3}\left(P, \Phi ; \tau_{1}, \tau_{2}\right)$ of $\tilde{x}_{3}\left(\rho, \phi ; \tau_{1}, \tau_{2}\right)$ w.r.t. $\rho$ and $\phi$ $\Rightarrow$ shift invariant w.r.t. $\rho$ and $\phi$.
4. Appropriate amplitude \& phase information of the transform $\rightarrow$ representation $F_{x}$.

## Properties of $F_{x}$ :

- Unique correspondence with class of images that are mutually related with SRS transformation.
- Noise insensitivity
- Stand-alone representation $\Rightarrow$ direct input to classifier.


## Feature size reduction

1. Reduced representation:

- amplitude information only, or
- zero-frequency Fourier coefficient only.

2. Reduced redundancy:

For each $\tau_{1} \in S^{\prime} \Rightarrow \tau_{2} \in S^{\prime \prime}\left(\tau_{1}\right)$

$$
\begin{aligned}
S^{\prime \prime}\left(\tau_{1}\right)= & \left\{\tau_{2} \in S^{\prime}: 0 \leq \tau_{1} \cdot \tau_{2} \leq\left|\tau_{1}\right|^{2},\right. \\
& \left.0 \leq \tau_{1} \cdot \tau_{2}^{\prime},\left(\tau_{1}-\tau_{2}\right) \in S^{\prime}\right\}
\end{aligned}
$$

3. Class parametrization:
$\left(\tau_{1}, \tau_{2}\right) \rightarrow\left(\theta_{1}, \theta_{2}\right) \quad \theta_{1}, \theta_{2} \in\left[0, \frac{\pi}{2}\right]$
$4-\mathrm{D}$ space $\rightarrow 2$-D space

The region $S^{\prime \prime}\left(\tau_{1}\right)$

$(-N+1,-N+1)$
( $\mathrm{N}-1,-\mathrm{N}+1$ )

## Discrete implementation

- Binary (black \& white) images: $x(\mathbf{t}) \in\{0,1\}$ Then:
- Image stored in binary integers
- Real multiplication $\Longleftrightarrow$ Boolean "AND".
- Discrete 2-D grid of $\left(\theta_{1}, \theta_{2}\right) \Rightarrow$ finite number of distinct classes $C\left(\theta_{1}, \theta_{2}\right)$.
- Discrete 2-D grid of $\rho$ and $\phi \Rightarrow$ discretization in the interior of each class.
- $\tilde{x}_{3}\left(\rho, \phi ; \theta_{1}, \theta_{2}\right) \rightarrow \tilde{X}_{3}\left(P, \Phi ; \theta_{1}, \theta_{2}\right)$
via FFT algorithm.
Parrallel implementation: possible.
- $\left|\tilde{X}_{3}\left(P, \Phi ; \theta_{1}, \theta_{2}\right)\right|$ : Final 4-D invariant representation.


## Algorithm

Step 1: Calculate all the horizontal shifts of the input image.

Step 2: For each $\tau_{1} \in S^{\prime}$ :

1. Find \& store all possible products $x(\mathbf{t}) x(\mathbf{t}+$ $\tau_{1}$ ), using the already calculated horizontal shifts and the logical operator AND.
2. Calculate \& quantize $\rho, \phi$ by comparing $\tau_{1}$ with $\tau_{o}=(1,0)$.
3. For each $\tau_{2} \in S^{\prime \prime}\left(\tau_{1}\right)$ :
(a) Find $x_{3}\left(\tau_{1}, \tau_{2}\right)$, using the products $x(\mathbf{t}) x\left(\mathbf{t}+\tau_{1}\right)$.
(b) If $x_{3}\left(\tau_{1}, \tau_{2}\right)=0$, proceed to the next $\tau_{2}$.
(c) Calculate $\&$ quantize $\theta_{1}, \theta_{2}$.
(d) $\tilde{x}_{3}\left(\rho, \phi ; \theta_{1}, \theta_{2}\right):=\tilde{x}_{3}\left(\rho, \phi ; \theta_{1}, \theta_{2}\right)+x_{3}\left(\tau_{1}, \tau_{2}\right)$

Step 3: Find $\tilde{X}_{3}\left(P, \Phi ; \theta_{1}, \theta_{2}\right)$; keep the amplitude of the transform only.

## Simulation Results

## Sample input images

## Invariant representations

# Invariant representations of scaled and rotated versions of the letter ' $N$ '. 

## Euclidean distances between invariant representations

