Imperial College of Science,
Technology & Medicine

INVESTIGATING THE CAPACITY OF A CELLULAR CDMA SYSTEM

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Code Division Multiple Access

- Accomplished by means of spread spectrum

- All users simultaneously use the entire spectrum devoted to the system

- Users are distinguished by means of different pseudo-noise (PN) sequences

Advantages of CDMA over FDMA and TDMA

- Higher Capacity
- Multipath Suppression
- Asynchronous operation
- Privacy
- Soft Capacity Limit
- Soft Hand-Off

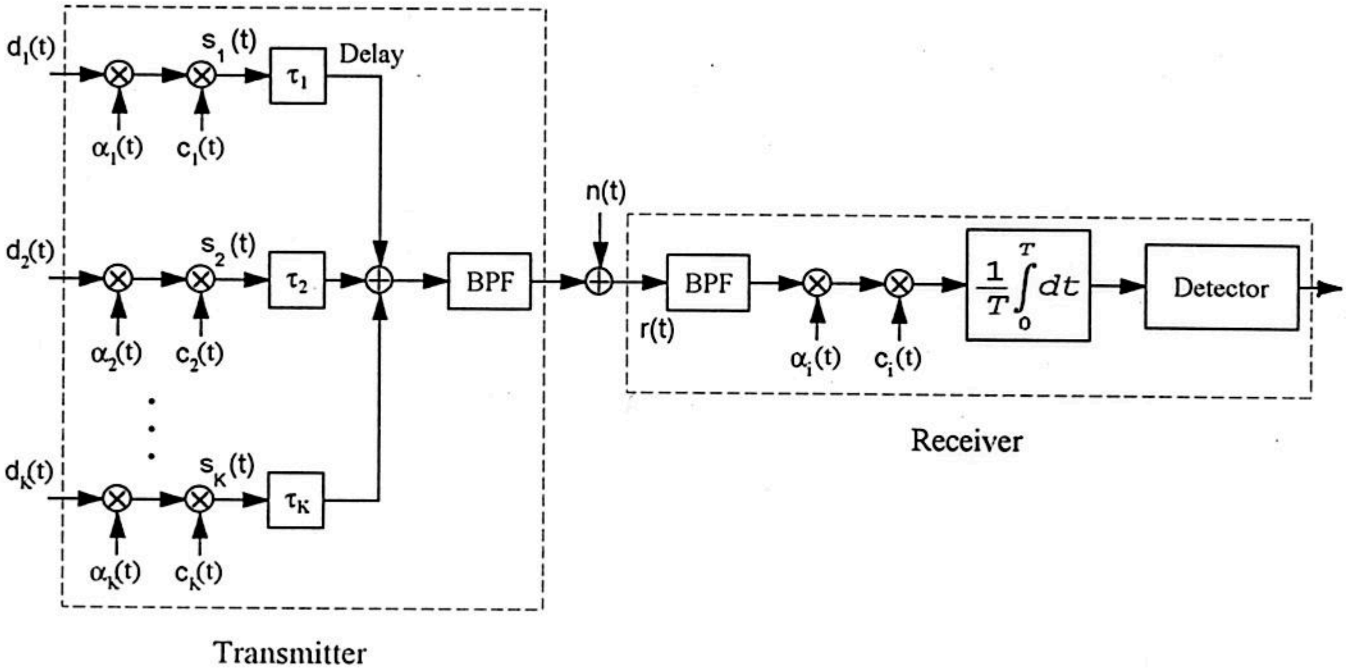
OBJECTIVES

- To study direct-sequence (DS) CDMA systems and find out how performance relates to number of users
  ⇒ Estimate capacity in a single-cell environment

- To calculate other-cell interference in a multiple-cell environment
  ⇒ Calculate forward & reverse link capacity

- To make a comparison with conventional techniques

Direct Sequence BPSK CDMA System



- Data signals: sequences of rectangular pulses of period T (bit period).
- Code waveforms: sequences of rectangular pulses of period T_c (chip period)
- Total bandwidth: $B_{ss} = 1/T_c$
- Asynchronous operation
- Coherent Receiver
- Gold Codes for PN sequences

Theoretical Results for Probability of Error

- Without Filters:

$$P_e = T\left\{\left(\frac{K-1}{3N} + \frac{N_0}{2E_b}\right)^{-1/2}\right\} = T\left\{\sqrt{SNR_{out}}\right\}$$

- With Filters:

$$P_e = T\left\{\left(\frac{K-1}{2N} + \frac{N_0}{2E_b}\right)^{-1/2}\right\} = T\left\{\sqrt{SNR_{out}}\right\}$$

where

$$T\{x\} = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-x^2/2} dx$$

K: Number of users

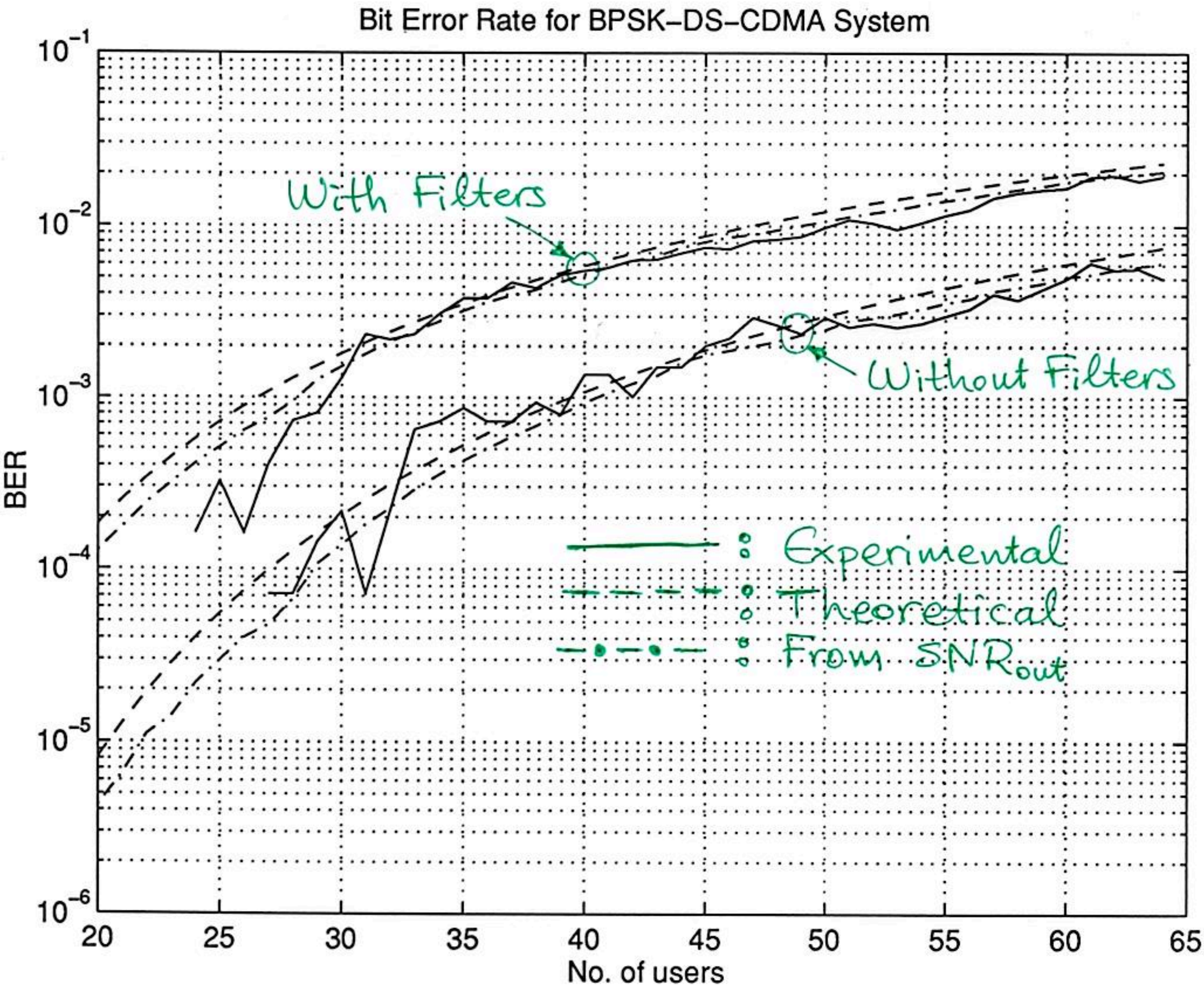
N: Number of chips / bit (processing gain, PG) equal to code period

E_b: Energy / bit

N_0: (one-sided) power spectral density of noise (white Gaussian)

SIMULATION RESULTS

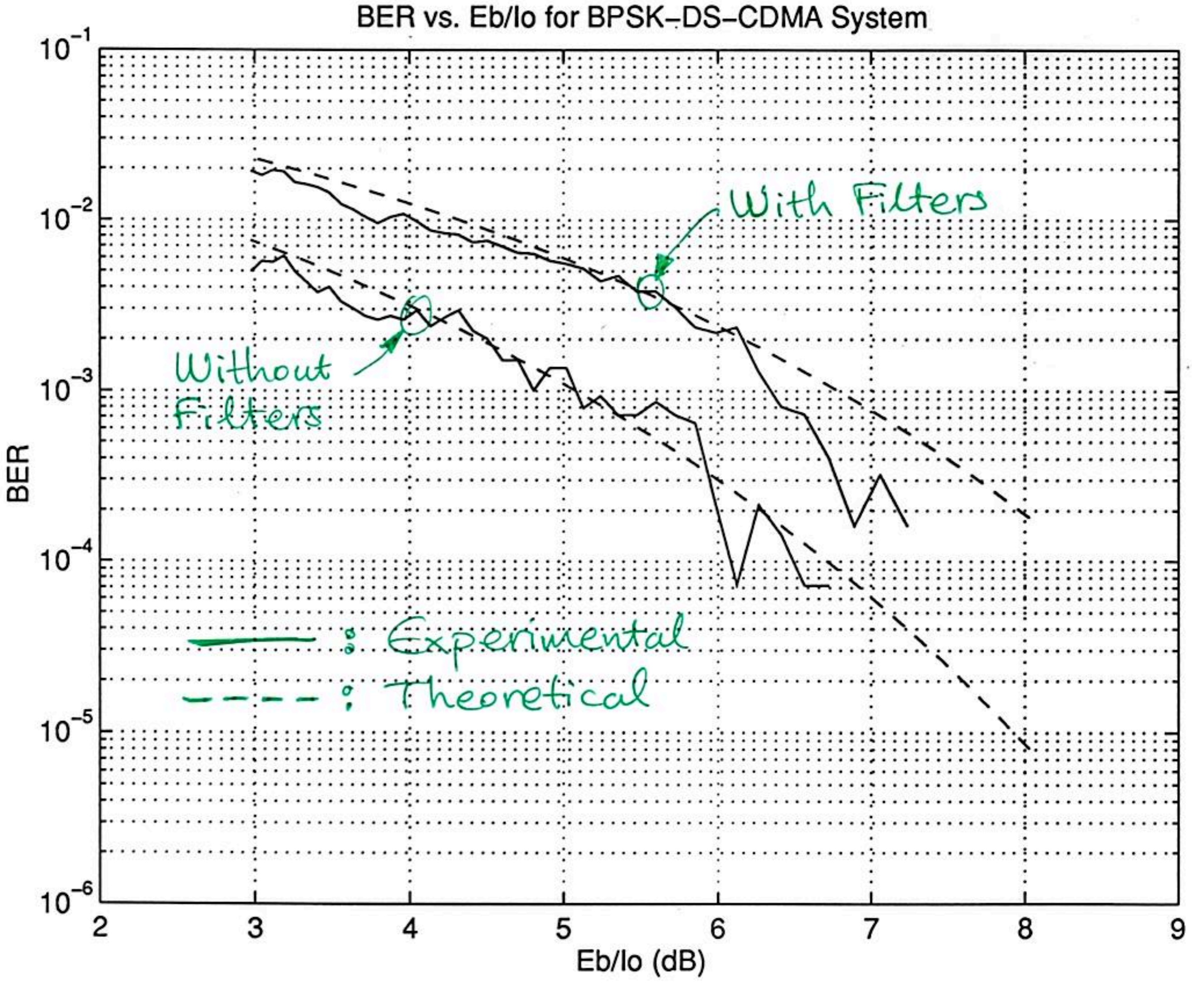
Probability of Error (Bit Error Rate) vs. Number of Users



- Processing Gain $PG = \frac{T}{T_c} = 127 = N$
- Total Bandwidth $B_{ss} = 1/T_c = 1.25$ MHz
- Bit Rate $R = 1/T = B_{ss}/127 \simeq 9800 \frac{\text{bits}}{\text{sec}}$
- Carrier Frequency $f_0 = 2B_{ss} = 2.5$ MHz
- Sampling Frequency $f_s = 20 f_0 = 50$ MHz

Probability of Error (BER) vs. E_b/I_o

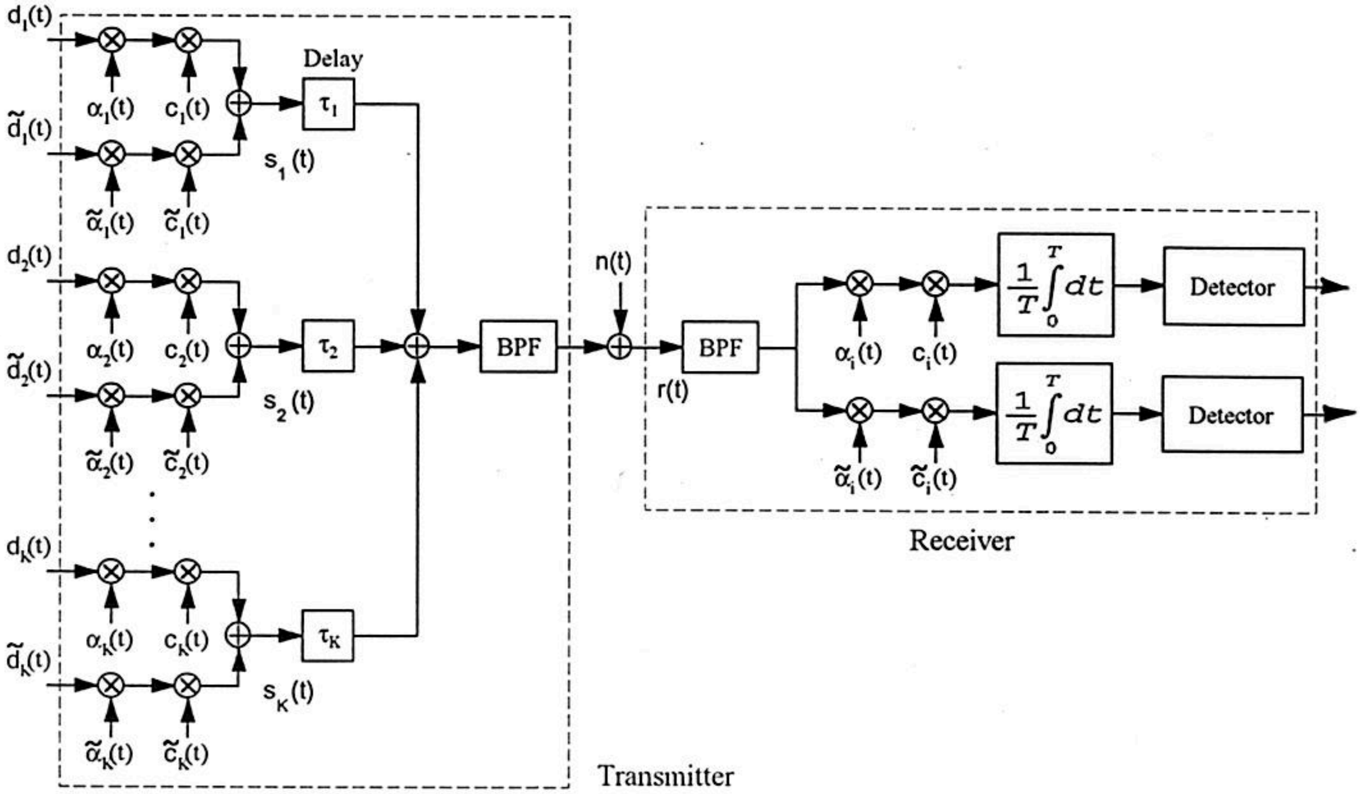
E_b: Signal Energy / Bit
I_o: Interference + Noise Power Spectral Density



$$\frac{E_b}{I_o} = \frac{E_b}{(K-1)PT_c + N_0} = \left[\frac{K-1}{N} + \frac{N_0}{E_b} \right]^{-1}$$

With Filters, $P_e = T\left\{ \sqrt{2E_b/I_o} \right\}$

Direct Sequence QPSK CDMA System



With same bandwidth and same symbol rate, QPSK system has

- Double bit rate
- Half processing gain

compared to BPSK system.

Theoretical Results for QPSK

- Without Filters:

$$P_e \cong T\left\{\left(\frac{2(K-1)}{3N}+\frac{N_0}{2E_b}\right)^{-1/2}\right\} = T\left\{\sqrt{SNR_{out}}\right\}$$

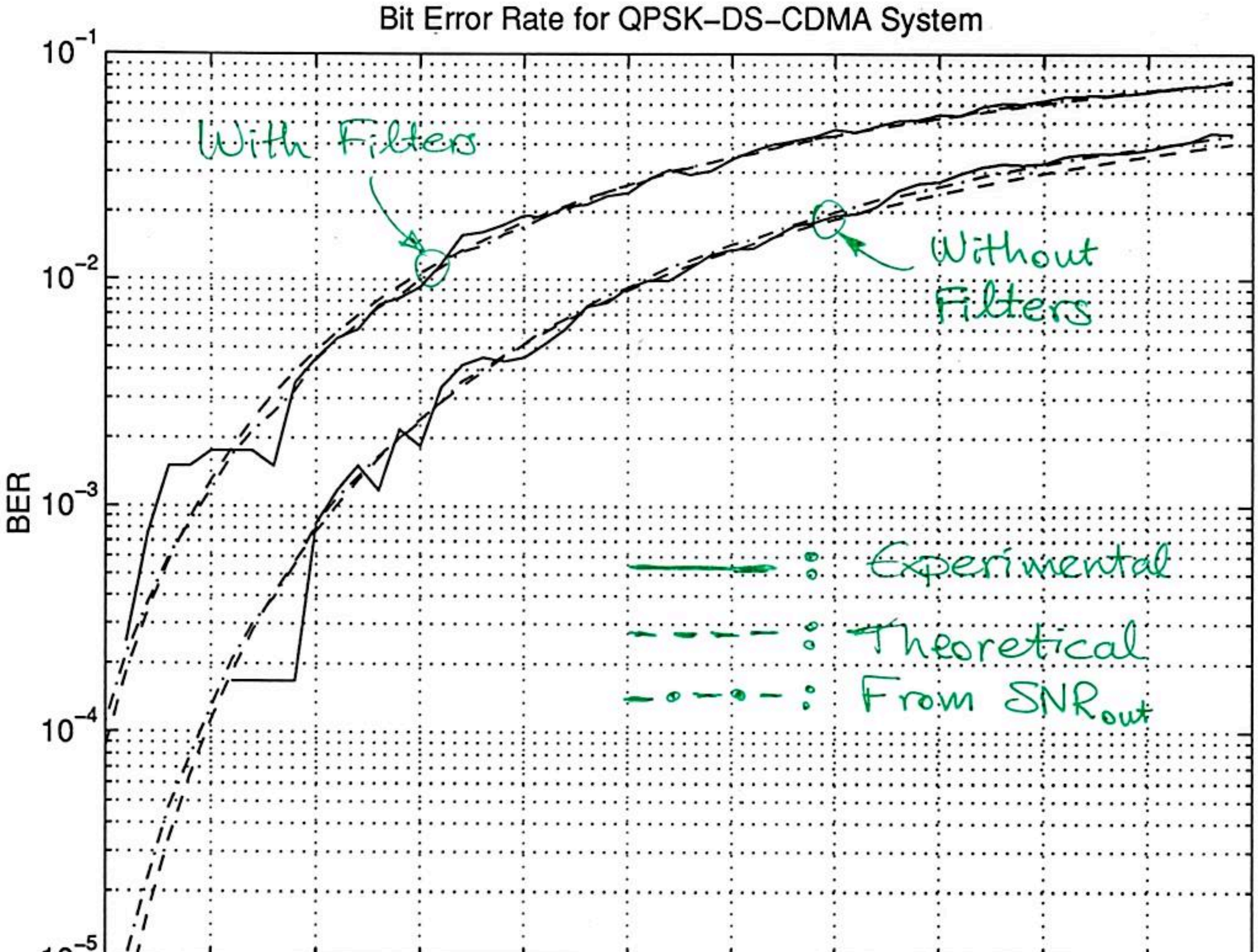
- With Filters:

$$P_e \cong T\left\{\left(\frac{K-1}{N}+\frac{N_0}{2E_b}\right)^{-1/2}\right\} = T\left\{\sqrt{SNR_{out}}\right\}$$

$\Rightarrow$ P_e of QPSK is equal to P_e of BPSK with twice the number of users

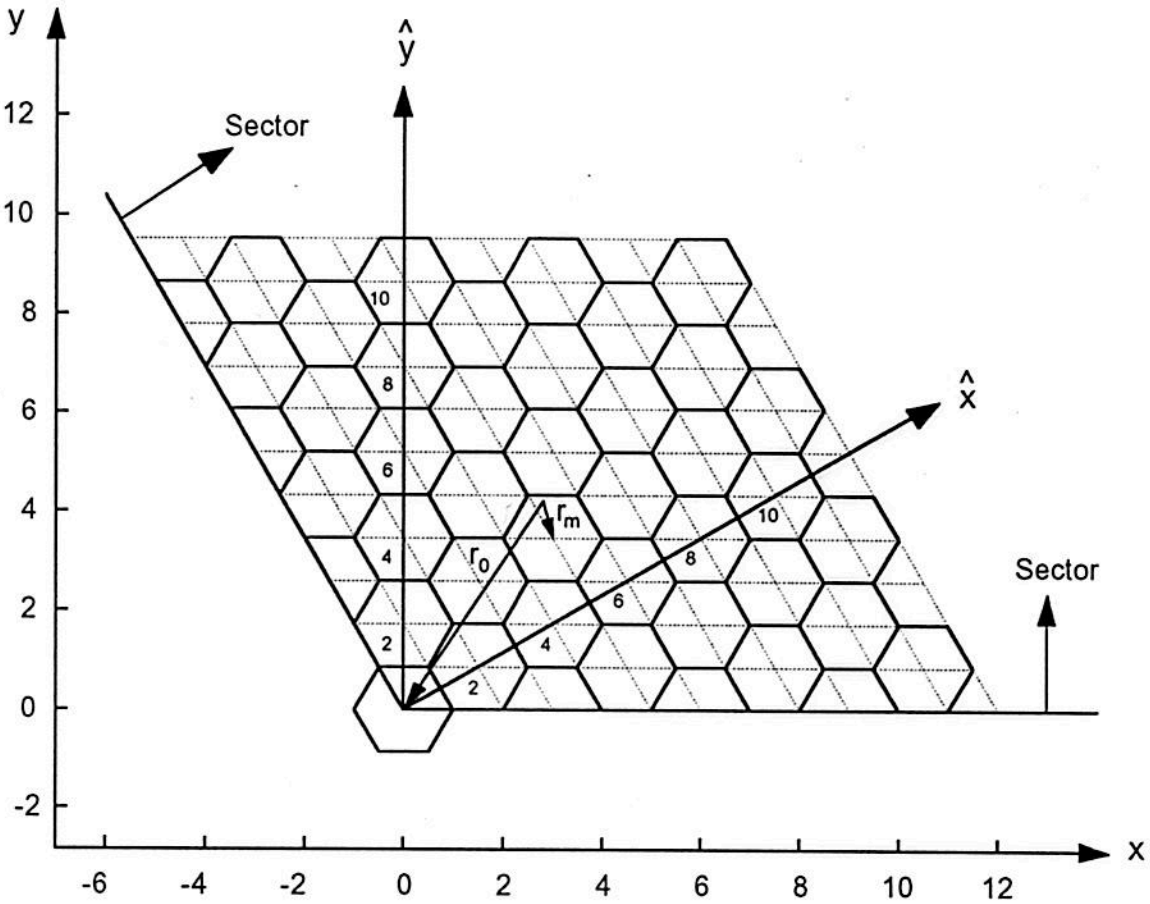
SIMULATION RESULTS

Probability of Error (BER) vs. Number of Users



- Bit Rate $R = 2/T = 2B_{ss}/127 \simeq 19600 \frac{bits}{sec}$
- Processing Gain $PG = \frac{B_{ss}}{2/T} = 63.5$

Multiple - Cell CDMA System



Ways of improving capacity

- Voice Activity Detection
- Sectorisation with directional antennas
- Reuse of the entire spectrum in all cells

Reverse Link Analysis

$$\frac{\mathcal{E}_b}{I_0} = \frac{P/R}{(K-1)P/B_{ss} + I/B_{ss} + N_0} = \frac{PG}{(K-1) + I/P + N/P} \geq \left(\frac{\mathcal{E}_b}{I_0}\right)_{min}$$

where

P : Signal Power

R : Bit Rate

K : Number of Users per cell.

B_{ss} : Total Bandwidth

N : Noise Power

N_0 : Noise Power Spectral Density

$\left(\frac{\mathcal{E}_b}{I_0}\right)_{min} = 7\,dB$ (5.01) for reverse link

With VAD & Sectorisation:

$$\frac{\mathcal{E}_b}{I_0} = \frac{PG}{\sum_{k=1}^{K_s-1} v_k + F + \eta} \geq \left(\frac{\mathcal{E}_b}{I_0}\right)_{min}$$

where

$K_s = K/3$: Number of users per sector

$\eta = N/P$: Inverse of signal-to-noise ratio

$F = I/P$: " " " " interference "

v_k : Voice activity RV (1 with prob. a, 0 with prob. $1-a$)

Other-Cell Interference

$$F = \iint_S \nu \left(\frac{r_c}{r_0}\right)^4 10^{x/10} \, L\left(\frac{r_c}{r_0}, x\right) \rho \, dx \, dy$$

where

S: sector area

$\rho = 2K_s/\sqrt{3}$: user density

x : zero mean Gaussian RV

$$L\left(\frac{r_c}{r_0}, x\right) = \begin{cases} 1, & \text{if } (r_c/r_0)^4 \, 10^{x/10} \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

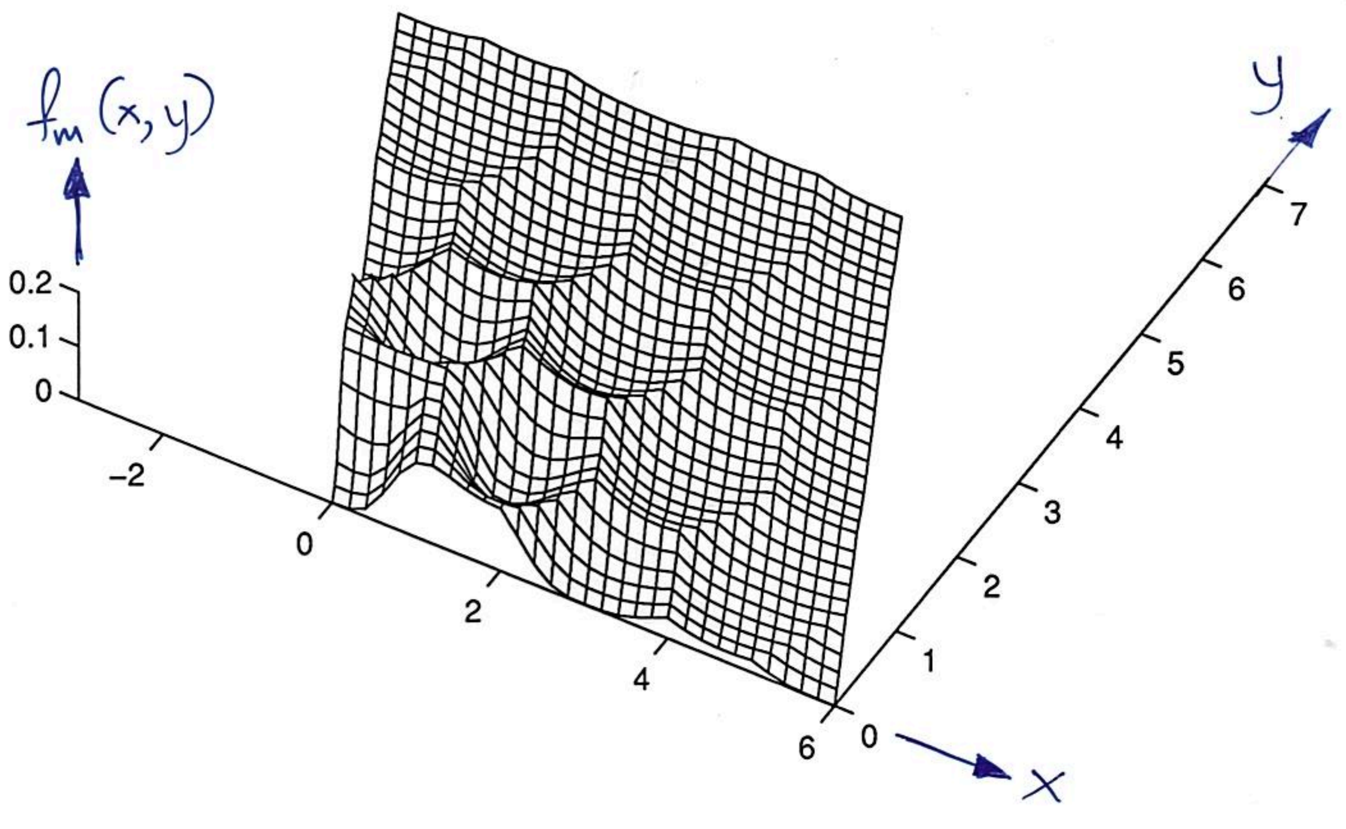
Analytical Solution:

F is a Gaussian RV with

$$m_F = E\{F\} = a \iint_S f_m(r_c/r_0) \rho \, dx dy = 0.42 \, K_s$$

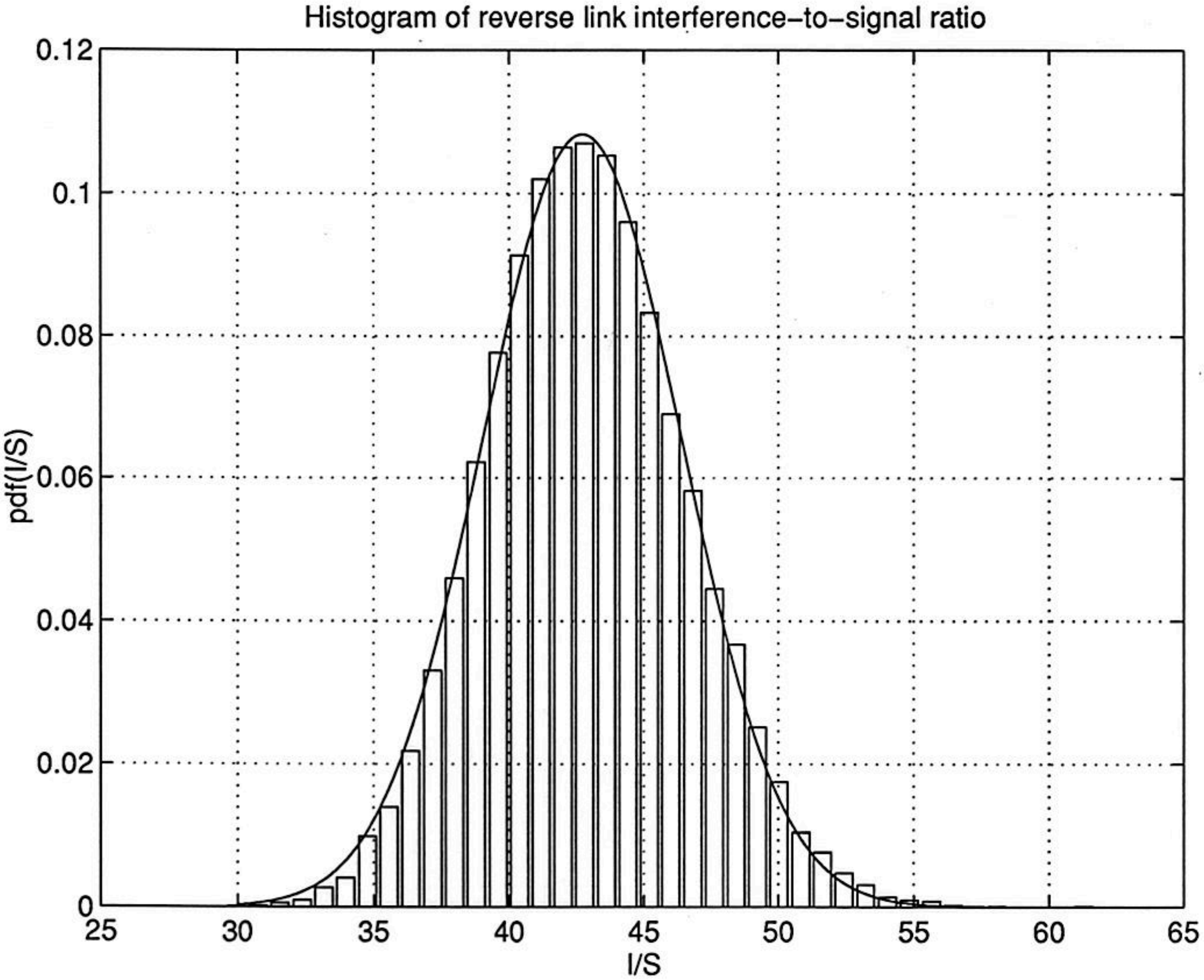
$$\sigma_F^2 = Var\{F\} = a \iint_S f_v(r_c/r_0) \rho \, dx dy = 0.13 \, K_s$$

Calculation of Other-Cell Interference



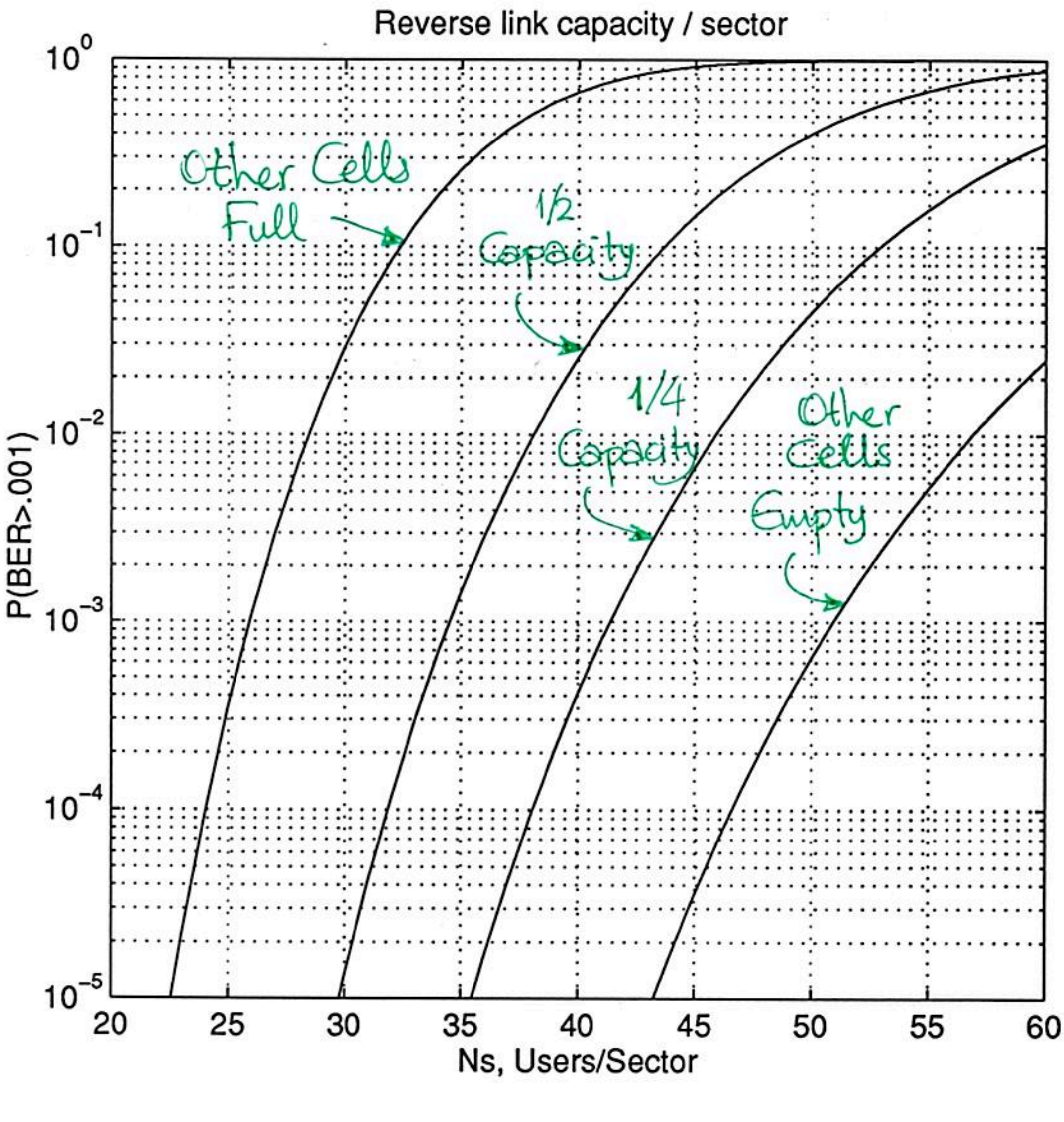
Monte-Carlo Simulation

with $K_s = 100$ users / sector



$$m_F = 42.71 \simeq 0.42\, K_s$$

$$\sigma_F^2 = 13.58 \simeq 0.13\, K_s$$



Reverse link capacity / sector
P(BER>.001)
Ns, Users/Sector
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
20
25
30
35
40
45
50
55
60
Other Cells Full
1/2 Capacity
1/4 Capacity
Other Cells Empty

Forward Link Analysis

$$\left(\frac{E_b}{I_0}\right)_i = \frac{c_i S_1 PG}{\sum_{j=1}^{M} S_j + n} \geq \left(\frac{E_b}{I_0}\right)_{min} \qquad (1)$$

where

$S_1 > S_2 > \cdots > S_M > 0$: received power from M nearest cell stations

c_i : fraction of cell site power devoted to user i

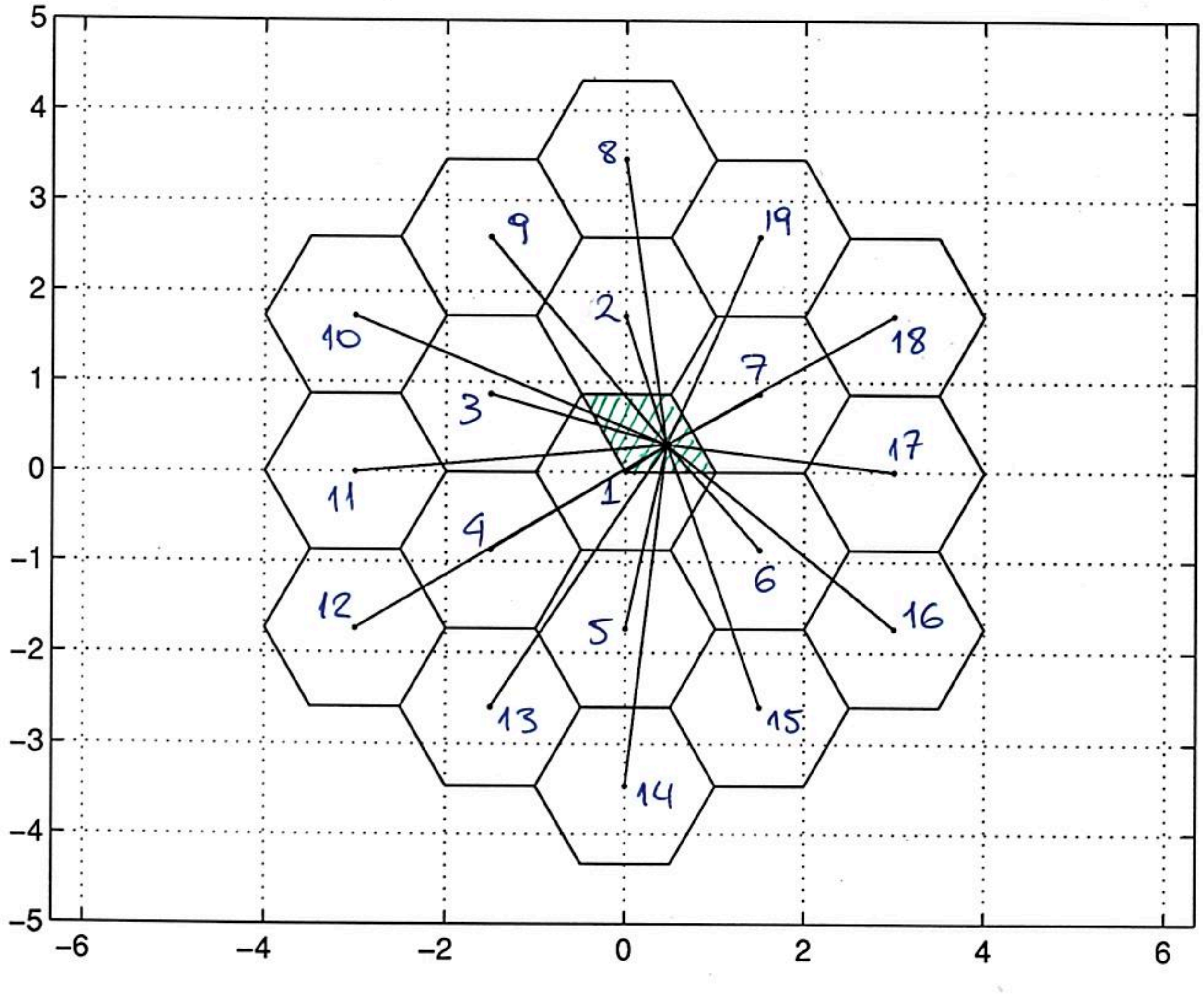
$$\sum_{i=1}^{K_s} c_i \leq 1 \qquad (2)$$

$$(1), (2) \Rightarrow \quad \sum_{i=1}^{K_s} f_i \leq \frac{PG}{(E_b/I_0)_{min}}$$

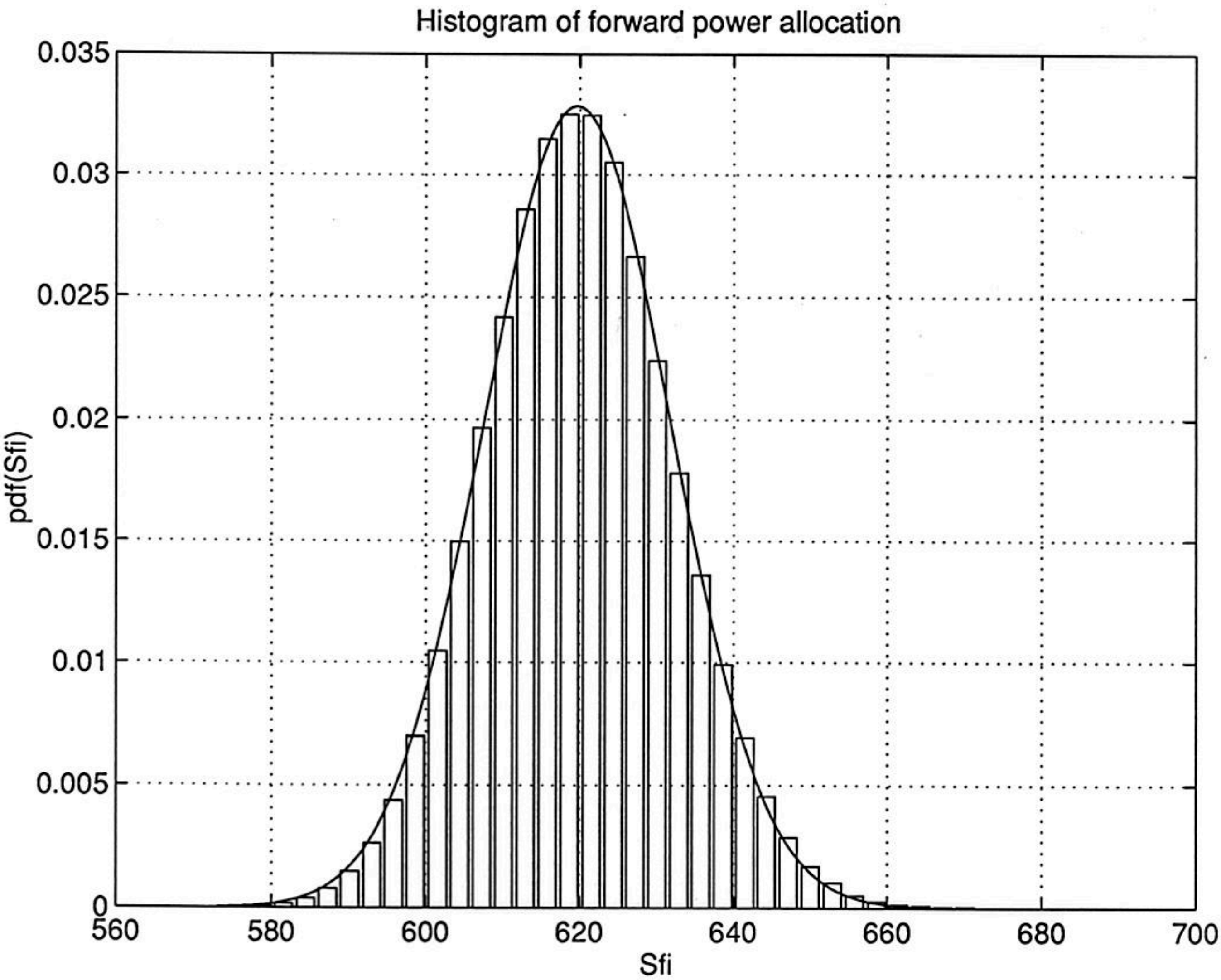
where $f_i = \left(\frac{\sum_{j=1}^{M} S_j}{S_1}\right)_i$

$\left(\frac{E_b}{I_0}\right)_{min} = 5\text{dB}$ (3.16) for forward link

Monte-Carlo Simulation for Forward Link

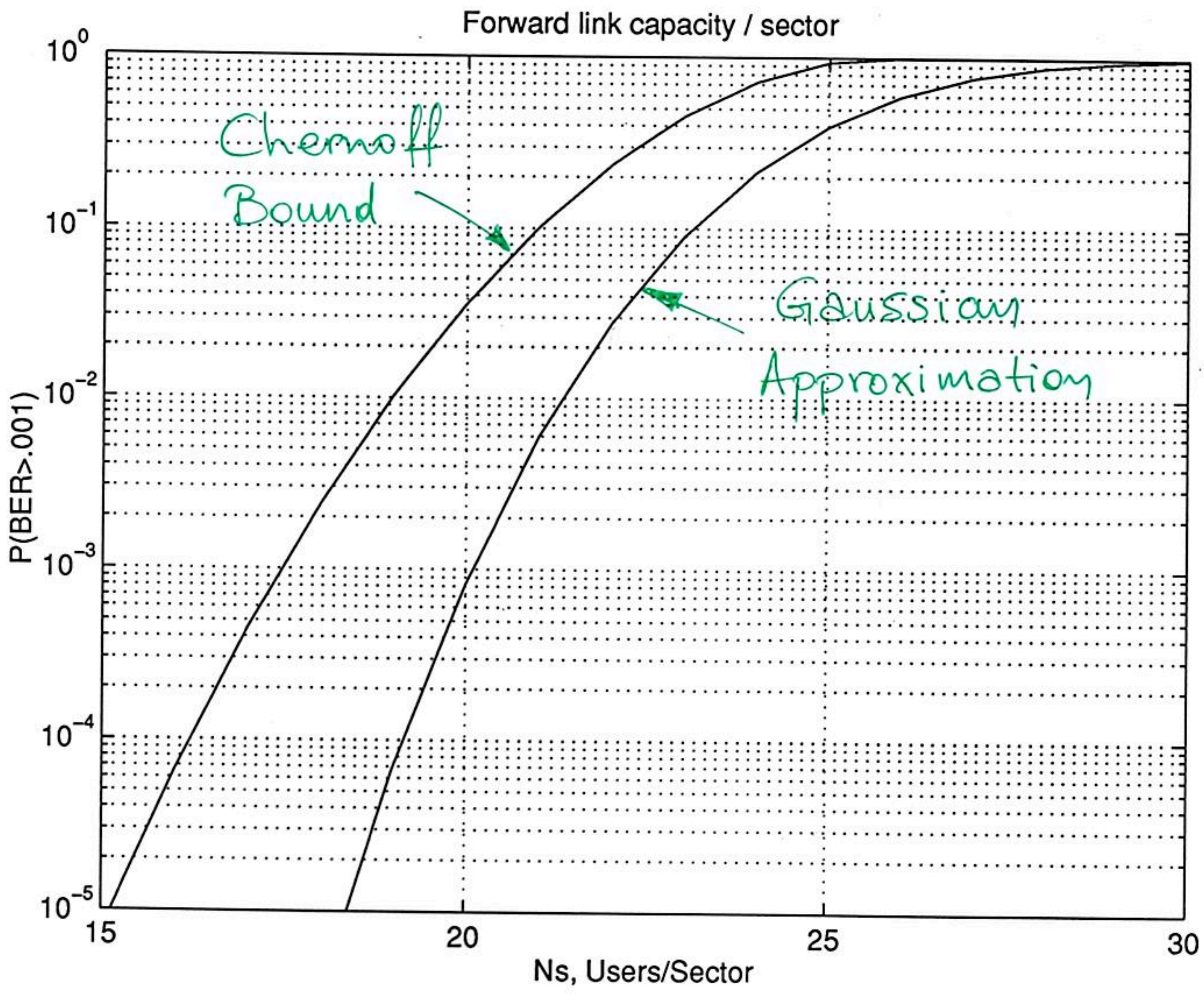


Results of Monte-Carlo Simulation for $K_s = 400$ users/sector



Mean: $619.6 \simeq 1.549\, K_s$

Variance: $147.4 \simeq 0.3685\, K_s$



Forward link capacity / sector
P(BER>.001)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
15
20
25
30
Ns, Users/Sector
Chernoff Bound
Gaussian Approximation

Comparison of CDMA with FDMA and TDMA

- CDMA Capacity:
  66 users/cell (forward link)
  84 users/cell (reverse link)
  Therefore, 660 users/cell for entire 12.5 MHz band

- Analog FM / FDMA Capacity:
  60 users/cell (12.5 MHz band)

- Digital FDMA / TDMA Capacity:
  180 users/cell (12.5 MHz band)

CONCLUSIONS

- CDMA Capacity 11 times greater than analog FM / FDMA capacity
- CDMA Capacity almost 4 times greater than digital FDMA/TDMA capacity.
- CDMA Capacity limited by forward link and not by reverse link.